

Review for Test 5.1 – 5.3, 5.6

Find the indefinite integral.

1) $\int (2x^2 + x - 1)dx$

$$\boxed{\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C}$$

2) $\int \frac{2}{\sqrt[3]{x}} dx$

$$\int 2x^{-\frac{1}{3}} dx$$

$$\boxed{3x^{\frac{2}{3}} + C}$$

3) $\int \frac{x^3 + 1}{x^2} dx$

$$\int [x + x^{-2}] dx$$

$$\frac{1}{2}x^2 - 1x^{-1} + C$$

$$\boxed{\frac{1}{2}x^2 - \frac{1}{x} + C}$$

4) $\int \frac{x^3 - 2x^2 + 1}{x^2} dx$

$$\int [x - 2 + x^{-2}] dx$$

$$\frac{1}{2}x^2 - 2x - x^{-1} + C$$

$$\boxed{\frac{1}{2}x^2 - 2x - \frac{1}{x} + C}$$

5) $\int (4x - 3 \sin x)dx$

$$\boxed{2x^2 + 3 \cos x + C}$$

6) $\int (5 \cos x - 2 \sec^2 x)dx$

$$\boxed{5 \sin x - 2 \tan x + C}$$

Find the general solution of the differential equation.

7) $\frac{dy}{dx} = 3x^2 dx$

$$\boxed{y = x^3 + C}$$

8) $\frac{dy}{dt} = t^{\frac{3}{2}}$

$$\boxed{y = \frac{2}{5}t^{\frac{5}{2}} + C}$$

Find the equation for y given the derivative and the indicated point on the curve.

9) $\frac{dy}{dx} = 2x - 1$; y passes through (1, 1)

$$y = x^2 - x + C$$

$$1 = 1 - 1 + C$$

$$C = 0$$

$$\boxed{y = x^2 - x + 1}$$

10) $\frac{dy}{dx} = x^2 - 1$; y passes through (-1, 3)

$$y = \frac{1}{3}x^3 - x + C$$

$$3 = -\frac{1}{3} + 1 + C$$

$$3 = \frac{2}{3} + C$$

$$\frac{7}{3} = C$$

$$\boxed{y = \frac{1}{3}x^3 - x + \frac{7}{3}}$$

11) A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.

a) Write the position function for the ball.

$$S(t) = -16t^2 + 96t + 0$$

b) Find the velocity function for the ball.

$$V(t) = -32t + 96$$

c) How long will it take the ball to rise to its maximum height?

vertex of $S(t)$ $(-\frac{b}{2a}, \text{ plug-in})$ $\frac{-96}{-32} = \frac{-48}{-16} = \frac{-24}{-8} = 3$

3 seconds

d) What is the maximum height?

$$S(3) = -16(3)^2 + 96(3) = 144 \quad \boxed{144 \text{ feet}}$$

e) When is the velocity of the ball one-half the initial velocity?

$$V(t) = 48$$

$$48 = -32t + 96 \quad -48 = -32t \quad t = 1.5 \quad \boxed{1.5 \text{ seconds}}$$

f) What is the height of the ball when its velocity is one-half the initial velocity?

$$S(1.5) = -16\left(\frac{9}{4}\right) + 96\left(\frac{3}{2}\right)$$

$$S(1.5) = -36 + 144 = 108$$

$$\frac{48}{3} \\ 144$$

108 feet

Solve the differential equations.

12) $f'(x) = 6x^2 \quad f(0) = -1$

$$\int f'(x) dx = f(x)$$

$$f(x) = \int 6x^2 dx$$

$$f(x) = 2x^3 + C \quad C = -1$$

$$\boxed{f(x) = 2x^3 - 1}$$

13) $f''(x) = 2 \quad f'(2) = 5 \quad f(2) = 10$

$$f'(x) = \int 2 dx$$

$$f'(x) = 2x + C$$

$$5 = 4 + C$$

$$C = 1$$

$$f'(x) = 2x + 1$$

$$f(x) = \int [2x+1] dx$$

$$f(x) = x^2 + x + C$$

$$10 = 4 + 2 + C$$

$$4 = C$$

$$\boxed{f(x) = x^2 + x + 4}$$

14) Use left and right Riemann sums to approximate the area of $y = \frac{10}{x^2 + 1}$ from $x = 0$ to $x = 2$, using 4 equal width rectangles.

interval	w	h
$(0, \frac{1}{2})$	$\frac{1}{2}$	10
$(\frac{1}{2}, 1)$	$\frac{1}{2}$	$10 \cdot \frac{4}{13} = 8$
$(1, \frac{3}{2})$	$\frac{1}{2}$	5
$(\frac{3}{2}, 2)$	$\frac{1}{2}$	$10 \cdot \frac{4}{13} = \frac{40}{13}$

$$\begin{aligned} & 23 + \frac{40}{13} \\ & \frac{299}{13} + \frac{40}{13} \\ & \frac{339}{13} [\frac{1}{2}] \end{aligned}$$

interval	w	h	
$(0, \frac{1}{2})$	$\frac{1}{2}$	8	$15 + \frac{40}{13}$
$(\frac{1}{2}, 1)$	$\frac{1}{2}$	5	$\frac{195}{13} + \frac{40}{13}$
$(1, \frac{3}{2})$	$\frac{1}{2}$	$\frac{40}{13}$	
$(\frac{3}{2}, 2)$	2	$\frac{235}{13} \cdot \frac{1}{2}$	

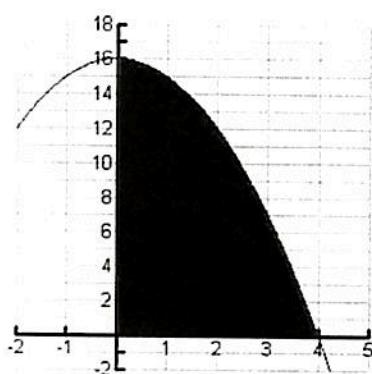
14) left sum: $\frac{339}{26} \approx 13.038$

right sum: $\frac{235}{26} \approx 9.038$

Find the area approximations for each function.

15) $y = -x^2 + 16$

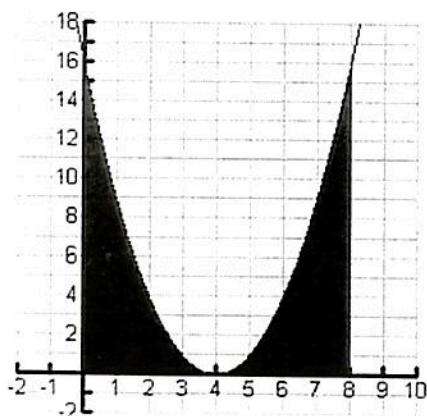
4 equal width trapezoids on the interval $[0, 4]$



Trapezoid Approximation:

Interval	h	$(b_1 + b_2)$	$\frac{1}{2}(b_1 + b_2)h$
$(0, 1)$	1	$16 + 15 = 31$	15.5
$(1, 2)$	1	$15 + 12 = 27$	13.5
$(2, 3)$	1	$12 + 7 = 19$	9.5
$(3, 4)$	1	$7 + 0 = 7$	3.5
total ...			$\frac{1}{2}(1)(84)$

16) $y = (x - 4)^2$ 4 equal width rectangles on the interval $[0, 8]$



Midpoint approximation:

Interval	w	h	Area
$(0, 2)$	2	9	18
$(2, 4)$	2	1	2
$(4, 6)$	2	1	2
$(6, 8)$	2	9	18

t (hours)	0	2	5	7	8	10
v(t) (miles per hour)	50	55	60	70	65	75

17) The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road.

a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure.

(remember: acceleration is the derivative (slope) of velocity)

acceleration \approx

5 miles/hour²

average rate of change from $t = 5$ to $t = 7$

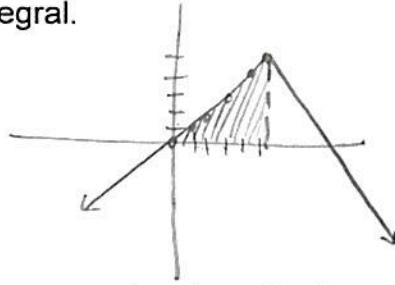
$$\frac{70 - 60}{7 - 5} = \frac{10}{2}$$

b) Use a right Riemann Sum with the subintervals given in the table to approximate $\int_0^{10} v(t) dt$. Indicate units of measure.

Interval	w	h	A
$(0, 2)$	2	55	110
$(2, 5)$	3	60	180
$(5, 7)$	2	70	140
$(7, 8)$	1	65	65
$(8, 10)$	2	75	150
total : 645			

Distance = $\int_0^{10} v(t) dt = \text{Area} \approx 645 \text{ miles}$

- 18) Sketch the region whose area is given by $\int_0^5 (5 - |x - 5|)dx$. Then use a geometric formula to evaluate the integral.



$$A = \frac{1}{2}(5)(5) = \frac{25}{2}$$

$$\boxed{\int_0^5 (5 - |x - 5|)dx = 12.5}$$

- 19) Find the area approximations for the table with 7 equal partitions.

a) midpoint =	<u>736</u>	interval	w
		(2, 6)	4
		(6, 10)	
		(10, 14)	
b) right sum =	<u>432</u>	(14, 18)	
		(18, 22)	
c) left sum =	<u>452</u>	(22, 26)	
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Name: _____

22) $\int (x^2 - 2)^2 dx =$

a) $\left(\frac{x^3}{3} - 2x\right)^2 + c$

b) $\frac{(x^2 - 2)^3}{6x} + c$

c) $\frac{2x}{3}(x^2 - 2)^3 + c$

d) $\frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$

e) $\left(\frac{x^2 - 2}{3}\right)^3 + c$

$$\int x^4 - 4x^2 + 4$$

23) Evaluate the given integral.

$$\int \frac{x^2 - 16}{x + 4} dx$$

$$\int \frac{(x+4)(x-4)}{(x+4)} dx$$

$$\int (x-4) dx$$

$$\boxed{\frac{1}{2}x^2 - 4x + C}$$

24) $\int \csc x (\cot x + \sin x) dx =$

a) $\sec x + \cos x + c$

d) $-\csc x + x + c$

b) $\csc x + x + c$

e) $-\sec x + \tan x + c$

c) $-\csc x + c$

$$\int [\csc x \cot x + 1] dx$$

$$-\csc x + x + C$$

25) In the graph below, the areas of regions A, B, and C are A = 3.2, B = 1.6, and C = 4.4.

$$\int_0^8 f(x) dx - 2 \int_0^8 dx$$

$$\left[-3.2 - 1.6 + 4.4 \right] - 16$$

$$-16.4$$

What is the value of $\int_0^8 (f(x) - 2) dx$?

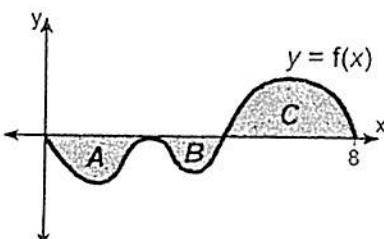
a) -2.4

b) -0.4

c) -16.4

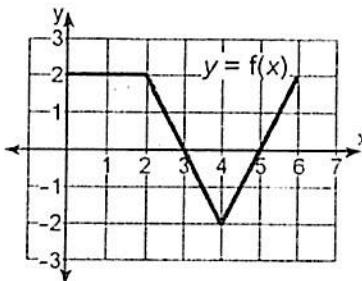
d) 16.4

e) -15.6



- 26) Function f is a piecewise function composed of three line segments, as shown below.

$$\int_1^3 f(x) dx + \int_3^5 f(x) dx + \int_5^6 f(x) dx \\ 3 + -2 + 1$$



What is the value of $\int_1^6 f(x) dx$?

a) 3

b) 7

c) 6

d) -2

e) 2

- 27) The table of values below represents a continuous function f .

interval	h	b_1+b_2	$\frac{1}{2}(b_1+b_2)h$
(1, 3)	2	60	60
(3, 4)	1	100	50
(4, 7)	3	110	165

x	$f(x)$
1	20
3	40
4	60
7	50

Using the subintervals [1,3], [3,4], and [4,7], what is the trapezoidal approximation of $\int_1^7 f(x) dx$?

a) 290

b) 270

c) 135

d) 275

e) 305

- 28) Given $\int_{-4}^1 f(x) dx = 3$ and $\int_1^3 f(x) dx = -5$. What is the value of $\int_3^{-4} f(x) dx$?

a) -2

b) 2

c) -8

d) 8

e) 3

$$\int_{-4}^3 f(x) dx = -2 \text{ So...}$$

- left! talk about in class
29) If three inscribed rectangles of uniform width are used to approximate $\int_{-2}^{-1} (x^2 + 4x + 6) dx$, then the approximation is

a) 52

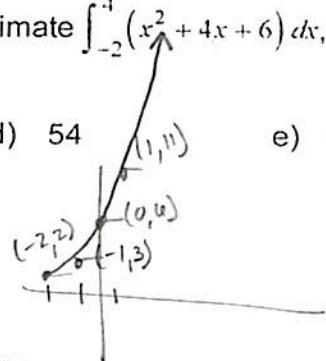
Interval	w	h	A
(-2, 0)	2	2	4
(0, 2)	2	4	12
(2, 4)	2	12	36

b) 78

c) 124

d) 54

e) 48



- 30) The table below gives various values of a function f on the closed interval $[0, 8]$.

x	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
f(x)	0.8	1.2	3.1	0.6	0.4	2.2	3.0	2.4	3.6

Using the given values and four subdivisions of width 2, the midpoint rule approximation of $\int_0^8 f(x) dx$ is

a) 12.4

b) 12.8

c) 11.8

d) 13.2

e) 12.6

Intervals	w	h	
(0, 2)	2	1.2	2.4
(2, 4)	2	0.6	1.2
(4, 6)	2	2.2	4.4
(6, 8)	2	2.4	4.8

