AP Calculus

Name KEY

Review for Test 5.1 – 5.3, 5.6

Find the indefinite integral.

1) $\int (2x^2 + x - 1)dx$

$\boxed{\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + c}$

2) $\int \frac{2}{\sqrt[3]{x}}dx$

$\int 2x^{-\frac{1}{3}}dx$

$\boxed{3x^{\frac{2}{3}} + c}$

3) $\int \frac{x^3 + 1}{x^2}dx$

$\int [x + x^{-2}]dx$

$\frac{1}{2}x^2 - 1x^{-1} + c$

$\boxed{\frac{1}{2}x^2 - \frac{1}{x} + c}$

4) $\int \frac{x^3 - 2x^2 + 1}{x^2}dx$

$\int [x - 2 + x^{-2}]dx$

$\frac{1}{2}x^2 - 2x - x^{-1} + c$

$\boxed{\frac{1}{2}x^2 - 2x - \frac{1}{x} + c}$

5) $\int (4x - 3\sin x)dx$

$\boxed{2x^2 + 3\cos x + c}$

6) $\int (5\cos x - 2\sec^2 x)dx$

$\boxed{5\sin x - 2\tan x + c}$

Find the general solution of the differential equation.

7) $\int \frac{dy}{dx} = \int 3x^2 dx$

$\boxed{y = x^3 + c}$

8) $\frac{dy}{dt} = t^{3/2}$

$\boxed{y = \frac{2}{5}t^{\frac{5}{2}} + c}$

Find the equation for y given the derivative and the indicated point on the curve.

9) $\frac{dy}{dx} = 2x - 1$; y passes though (1, 1)

$y = x^2 - x + c$

$1 = 1 - 1 + c$

$1 = c$

$\boxed{y = x^2 - x + 1}$

10) $\frac{dy}{dx} = x^2 - 1$; y passes through (-1, 3)

$y = \frac{1}{3}x^3 - x + c$

$3 = -\frac{1}{3} + 1 + c$

$3 = \frac{2}{3} + c$

$\frac{7}{3} = c$

$\boxed{y = \frac{1}{3}x^3 - x + \frac{7}{3}}$

11) A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.

a) Write the position function for the ball.

$s(t) = -16t^2 + 96t + 0$

b) Find the velocity function for the ball.

$v(t) = -32t + 96$

c) How long will it take the ball to rise to its maximum height?

vertex of $s(t)$ $\left(\frac{-b}{2a}, \text{plug-in}\right)$ $\frac{-96}{-32} = \frac{-48}{-16} = \frac{-24}{-8} = 3$

$\boxed{3 \text{ seconds}}$

d) What is the maximum height?

$s(3) = -16(3)^2 + 96(3) = 144$ $\boxed{144 \text{ feet}}$

e) When is the velocity of the ball one-half the initial velocity?

$V(t) = 48$

$48 = -32t + 96$ $-48 = -32t$ $t = 1.5$ $\boxed{1.5 \text{ seconds}}$

f) What is the height of the ball when its velocity is one-half the initial velocity?

$s(1.5) = -16\left(\frac{9}{4}\right) + 96\left(\frac{3}{2}\right)$

$s(1.5) = -36 + 144 = 108$

48 × 3 = 144

$\boxed{108 \text{ feet}}$

Solve the differential equations.

12) $f'(x) = 6x^2$ $f(0) = -1$

$\int f'(x)\,dx = f(x)$

$f(x) = \int 6x^2\,dx$

$f(x) = 2x^3 + c$ $c = -1$

$\boxed{f(x) = 2x^3 - 1}$

13) $f''(x) = 2$ $f'(2) = 5$ $f(2) = 10$

$f'(x) = \int 2\,dx$

$f'(x) = 2x + c$

$5 = 4 + c$

$c = 1$

$f'(x) = 2x + 1$

$f(x) = \int [2x+1]\,dx$

$f(x) = x^2 + x + c$

$10 = 4 + 2 + c$

$4 = c$

$\boxed{f(x) = x^2 + x + 4}$

14) Use left and right Riemann sums to approximate the area of $y = \frac{10}{x^2+1}$ from x = 0 to x = 2, using 4 equal width rectangles.

interval	w	h
$(0, \frac{1}{2})$	$\frac{1}{2}$	10
$(\frac{1}{2}, 1)$		$10 \cdot \frac{4}{5} = 8$
$(1, \frac{3}{2})$		5
$(\frac{3}{2}, 2)$		$10 \cdot \frac{4}{13} = \frac{40}{13}$

$23 + \frac{40}{13}$

$\frac{299}{13} + \frac{40}{13}$

$\frac{339}{13}\left[\frac{1}{2}\right]$

23 × 13 = 69 + 230 = 299

interval	w	h
$(0, \frac{1}{2})$	$\frac{1}{2}$	8
$(\frac{1}{2}, 1)$		5
$(1, \frac{3}{2})$		$\frac{40}{13}$
$(\frac{3}{2}, 2)$		2

$15 + \frac{40}{13}$

$\frac{195}{13} + \frac{40}{13}$

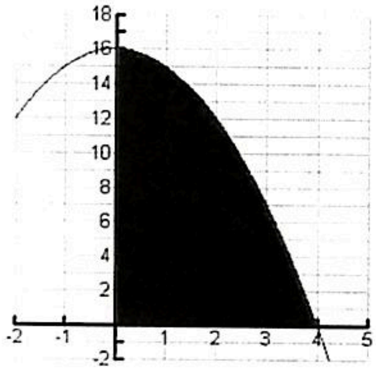
$\frac{235}{13} \cdot \frac{1}{2}$

15 × 13 = 45 + 150 = 195

14) left sum: $\frac{339}{26} \approx 13.038$ right sum: $\frac{235}{26} \approx 9.038$

Find the area approximations for each function.

15) $y = -x^2 + 16$ 4 equal width trapezoids on the interval [0, 4]

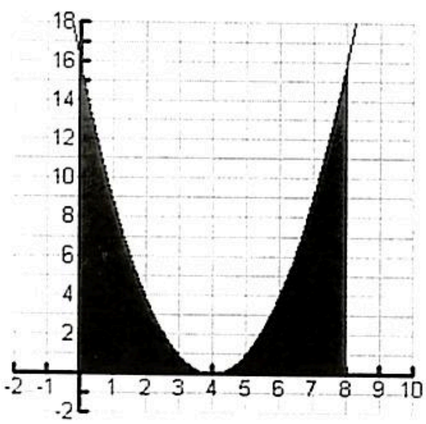


Trapezoid Approximation: 42

interval	h	(b_1+b_2)	$\frac{1}{2}(b_1+b_2)h$
(0,1)	1	16+15 = 31	15.5
(1,2)		15+12 = 27	13.5
(2,3)		12+7 = 19	9.5
(3,4)		7+0 = 7	3.5

total... $\frac{1}{2}(1)(84)$

16) $y = (x-4)^2$ 4 equal width rectangles on the interval [0, 8]



Midpoint approximation: 40

interval	w	h	Area
(0,2)	2	9	18
(2,4)	2	1	2
(4,6)	2	1	2
(6,8)	2	9	18

t (hours)	0	2	5	7	8	10
v(t) (miles per hour)	50	55	60	70	65	75

17) The table above gives the velocity v(t) at selected times t of a car traveling along a straight road.

a) Use the values of the table to approximate the acceleration of the car at time t = 6. Show the work that leads to your answer and indicate units of measure.

(remember: acceleration is the derivative (slope) of velocity)

acceleration ≈ average rate of change from t=5 to t=7 $\frac{70-60}{7-5} = \frac{10}{2}$

$\boxed{5 \text{ miles/hour}^2}$

b) Use a right Riemann Sum with the subintervals given in the table to approximate $\int_0^{10} v(t)dt$. Indicate units of measure.

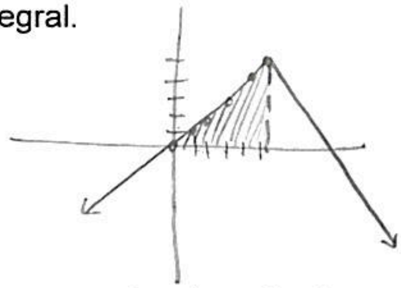
area

Distance $= \int_0^{10} v(t)dt = \text{Area} \approx \boxed{645 \text{ miles}}$

interval	w	h	A
(0,2)	2	55	110
(2,5)	3	60	180
(5,7)	2	70	140
(7,8)	1	65	65
(8,10)	2	75	150

total: 645

18) Sketch the region whose area is given by $\int_0^5 (5-|x-5|)dx$. Then use a geometric formula to evaluate the integral.



$A = \frac{1}{2}(5)(5) = \frac{25}{2}$

$\boxed{\int_0^5 (5-|x-5|)dx = 12.5}$

19) Find the area approximations for the table with 7 equal partitions.

a) midpoint= 736

$4[27+13+43+30+50+11+10]$

b) right sum= 432

$4[1+5+31+21+7+39+4]$

c) left sum= 452

$4[9+1+5+31+21+7+39]$

interval	w
(2, 6)	4
(6, 10)	
(10, 14)	
(14, 18)	
(18, 22)	
(22, 26)	
(26, 30)	

x	y
2	9
4	27
6	1
8	13
10	5
12	43
14	31
16	30
18	21
20	50
22	7
24	11
26	39
28	10
30	4

20) Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, evaluate each of the following.

a) $\int_0^6 f(x)dx = 3$

b) $\int_6^3 f(x)dx = 1$

c) $\int_4^4 f(x)dx = 0$

d) $\int_3^6 -10f(x)dx$

$-10\int_3^6 f(x)dx = 10$

Calculator

21) Use the Trapezoidal Rule with n = 4 to approximate the definite integral $\boxed{\int_1^2 \frac{1}{1+x^3}dx \approx 0.721}$

interval	h	b_1+b_2	$\frac{1}{2}(b_1+b_2)h$
$(1, \frac{1}{4})$	$\frac{1}{4}$	$\frac{1}{2}+\frac{64}{65}$	$\frac{193}{1040}$
$(\frac{1}{4}, \frac{1}{2})$		$\frac{64}{65}+\frac{8}{9}$	$\frac{137}{585}$
$(\frac{1}{2}, \frac{3}{4})$		$\frac{8}{9}+\frac{64}{91}$	$\frac{163}{819}$
$(\frac{3}{4}, 2)$		$\frac{64}{91}+\frac{1}{9}$	$\frac{667}{6552}$

Name: ______________________________

22) $\int (x^2 - 2)^2 \, dx =$

a) $\left(\frac{x^3}{3} - 2x\right)^2 + c$ b) $\frac{(x^2 - 2)^3}{6x} + c$ c) $\frac{2x}{3}(x^2 - 2)^3 + c$

d) $\frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$ e) $\left(\frac{x^2 - 2}{3}\right)^3 + c$

$\int x^4 - 4x^2 + 4$

23) Evaluate the given integral.

$\int \frac{x^2 - 16}{x + 4} \, dx$

$\int \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}} dx$

$\int (x-4) dx$

$\boxed{\frac{1}{2}x^2 - 4x + c}$

24) $\int \csc x (\cot x + \sin x) \, dx =$

a) $\sec x + \cos x + c$ b) $\csc x + x + c$ c) $-\csc x + c$

d) $-\csc x + x + c$ e) $-\sec x + \tan x + c$

$\int [\csc x \cot x + 1] dx$

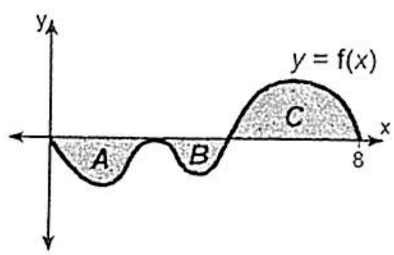
$-\csc x + x + c$

25) In the graph below, the areas of regions A, B, and C are $A = 3.2$, $B = 1.6$, and $C = 4.4$.

$\int_0^8 f(x) dx - 2\int_0^8 dx$

$[-3.2 - 1.6 + 4.4] - 16$

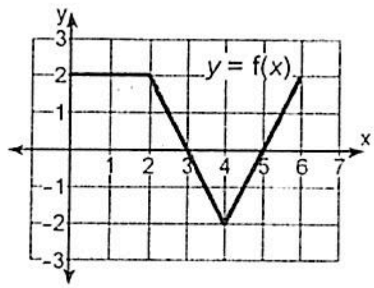
-16.4



What is the value of $\int_0^8 (f(x) - 2) \, dx$?

a) -2.4 b) -0.4 c) -16.4 d) 16.4 e) -15.6

26) Function f is a piecewise function composed of three line segments, as shown below.



$\int_1^3 f(x)dx + \int_3^5 f(x)dx + \int_5^6 f(x)dx$

$3 + -2 + 1$

What is the value of $\int_1^6 f(x)\,dx$?

a) 3 b) 7 c) 6 d) -2 e) 2

27) The table of values below represents a continuous function f.

interval	h	b_1+b_2	$\frac{1}{2}(b_1+b_2)h$
(1,3)	2	60	60
(3,4)	1	100	50
(4,7)	3	110	165

x	f(x)
1	20
3	40
4	60
7	50

Using the subintervals [1,3], [3,4], and [4,7], what is the trapezoidal approximation of $\int_1^7 f(x)\,dx$?

a) 290 b) 270 c) 135 d) 275 e) 305

28) Given $\int_{-4}^{1} f(x)\,dx = 3$ and $\int_1^3 f(x)\,dx = -5$. What is the value of $\int_3^{-4} f(x)\,dx$?

a) -2 b) 2 c) -8 d) 8 e) 3

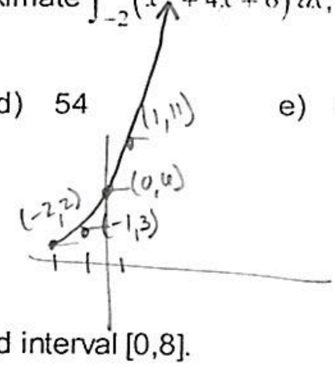
$\int_{-4}^{3} f(x)\,dx = -2$ so...

left! talk about in class

29) If three inscribed rectangles of uniform width are used to approximate $\int_{-2}^{4}\left(x^2+4x+6\right)dx$, then the approximation is

a) 52 b) 78 c) 124 d) 54 e) 48

Interval	w	h	A
(-2,0)	2	2	4
(0,2)		6	12
(2,4)		18	36



30) The table below gives various values of a function f on the closed interval [0,8].

x	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
f(x)	0.8	1.2	3.1	0.6	0.4	2.2	3.0	2.4	3.6

Using the given values and four subdivisions of width 2, the midpoint rule approximation of $\int_0^8 f(x)\,dx$ is

a) 12.4 b) 12.8 c) 11.8 d) 13.2 e) 12.6

Intervals	w	h	
(0,2)	2	1.2	2.4
(2,4)		0.6	1.2
(4,6)		2.2	4.4
(6,8)		2.4	4.8