AP Calculus AB Name Key

Review for Test – Limits

No Calculator!

Find the limit (if it exists).

1) $\lim_{x\to 4}\sqrt{x+2}$ $=\sqrt{6}$

2) $\lim_{x\to -2}\frac{\cancel{x+2}}{x^2-4}$ $= -\frac{1}{4}$

$(\cancel{x+2})(x-2)$

$(\cancel{x+5})(x^2-5x+25)$

3) $\lim_{x\to -5}\frac{x^3+125}{\cancel{x+5}}$ $=75$

4) $\lim_{x\to \frac{\pi}{4}}\frac{4x}{\tan x}$ $=\frac{\pi}{1}=\pi$

5) $\lim_{x\to 1^-} g(x)$, where $g(x)=\begin{cases}\sqrt{1-x}, & x\le 1\\ x+1, & x>1\end{cases}$

6) $\lim_{t\to 1} h(t)$, where $h(t)=\begin{cases}t^3+1, & t<1\\ \frac{1}{2}(t+1), & t\ge 1\end{cases}$

Determine the intervals on which the function is continuous.

7) $f(x)=\begin{cases}5-x, & x\le 2\\ 2x-3, & x>2\end{cases}$

1 $f(2)=3$ not continuous @ $x=2$

2 $\lim_{x\to 2^-} f(x)=1$ $\lim_{x\to 2^+} f(x)=3$ $\lim_{x\to 2^-} f(x)\ne\lim_{x\to 2^+}$

$(-\infty,2)\cup(2,\infty)$

8) $f(x)=\frac{1}{(x-2)^2}$ VA $x=2$

$(-\infty,2)\cup(2,\infty)$

9) Determine the value of c such that the function is continuous on the entire real line.

$f(x)=\begin{cases}x+3, & x\le 2\\ cx+6, & x>2\end{cases}$

you need $x=2$ to be Continuous

$x+3=cx+6$ @ $x=2$

$5=2c+6$

$-1=2c$

$c=-\frac{1}{2}$

10) Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 3$ has a zero in the interval [1, 2].

$f(1)=-1$ $f(2)=5$

because $f(x)$ goes from $-$ to $+$ must be a zero

Find the vertical asymptotes (if any) of the graphs of the function.

11) $f(x)=\frac{8}{(x-10)^2}$ $x=10$

12) $g(x)=\frac{x^2-49}{\cancel{x-7}}$ with $(x+7)(\cancel{x-7})$

none

Find the one-sided limit.

13) $\lim_{x\to -2^-}\frac{2x^2+x+1}{x+2}$ $\to\frac{11}{4}$

14) $\lim_{x\to -1^+}\frac{\cancel{x+1}}{x^3+1}$ $=\frac{1}{3}$

$(\cancel{x+1})(x^2-x+1)$

Find the limit.

15) $\lim_{x\to\infty}\frac{2x^2}{3x^2+5}$ HA: $y=\frac{2}{3}$ $=\frac{2}{3}$

16) $\lim_{x\to\infty}\frac{2x}{3x^2+5}$ $=0$

17) $\lim_{x\to -\infty}\frac{3x^2}{x+5}$ DNE no HA so… $-\infty$

Find any vertical and horizontal asymptotes of the graph of the function.

18) $h(x)=\frac{2x+3}{x-4}$ HA: $y=2$ VA: $x=4$

19) $g(x)=\frac{5x^2}{x^2+2}$ HA: $y=5$ VA: none

20) Let f(x) be a function defined by $f(x)=\begin{cases}\cos x, & x\le 0\\ x^2+1, & x>0\end{cases}$. Show that f(x) is continuous at x = 0. (You must use the 3 step process discussed in the day 2 notes on limits!)

1 $f(0)=1$

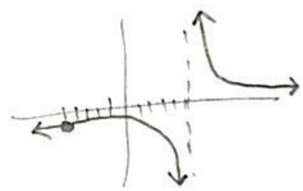
2 $\lim_{x\to 0^-} f(x)=1$ $\lim_{x\to 0^+} f(x)=1$ therefore $\lim_{x\to 0} f(x)=1$

3 b/c $f(0)=\lim_{x\to 0} f(x)$ then $f(x)$ is continuous @ $x=0$

21) The $\lim_{x\to 4} x^2 - 5x + 9$ is 5

22) The $\lim_{x\to 5^-} \frac{\cancel{x+5}}{x^2 - 25}$ is $\quad \lim_{x\to 5^-} \frac{x+5}{x^2-25} = -\infty$

$(\cancel{x+5})(x-5)$



23) $\lim_{x\to 0} \frac{-5x^5 + 3x^3}{x}$ is $\quad \lim_{x\to 0} -5x^4 + 3x^2$

$= 0$

24) $\lim_{x\to\infty} \frac{4x^2 + 16}{x^3 - 64}$ is $= 0$

$4(x^2+4)$

$(x-4)(x^2+4x+16)$

25) $\lim_{x\to\infty} \frac{3 - 5x^2 - 2x^3}{6x^3 + x^2 - 2x + 1}$ is $= \frac{-1}{3}$

26) The graph of $f(x) = \frac{ax + 10}{x - b}$ has $x = -5$ and $y = 3$ as asymptotes. What is the value of $a - b$?

$b = -5 \quad a = 3$

27) $\lim_{x\to \frac{\pi}{4}} (\cos^2 x - 1)$ is $= \frac{-1}{2}$

D 28) The graph of which of the following functions has $y = -2$ as a horizontal asymptote?

a) $f(x) = \frac{|x - 2|}{x + 2}$ b) $f(x) = \frac{x^2}{x^2 - 4}$ c) $f(x) = \frac{x^2 - 4}{2x^2}$

d) $f(x) = \frac{2x^2}{4 - x^2}$ e) $f(x) = \frac{2x^2}{4 + x^2}$

E 29) Which of the following statements is true for the graph of $f(x) = \frac{3\cancel{x}}{x^3 - 9x}$?

$x(x^2-9)$

$\cancel{x}(x-3)(x+3)$

~~a)~~ $\cancel{x = 0}$, $x = 3$, and $x = -3$ are vertical asymptotes.

~~b)~~ $y = \frac{1}{3}$ is a horizontal asymptote. $y = 0$

~~c)~~ $x = 3$ is the only vertical asymptote. $x = -3$

~~d)~~ The graph of function f has no horizontal asymptote. $y = 0$

e) $x = 3$ and $x = -3$ are vertical asymptotes.

D 30) Given function f defined below.

$x(x+2)$

$$f(x)=\begin{cases} x^2+2x, & x \le a \\ x+2, & x > a \end{cases}$$

$x^2+2x = x+2$

$x^2+x-2=0$

$(x+2)(x-1)=0$

Determine all values of a for which $\lim_{x \to a} f(x)$ exists.

a) $a = -2$
b) $a = 1$
c) $a = 2$ and $a = -1$
d) $a = -2$ and $a = 1$
e) No value of a exists.

C 31) Given function f as defined below.

$x(x^2-1) = \frac{\cancel{x}(x-1)(x+1)}{\cancel{x}}$

$$f(x)=\begin{cases} \dfrac{x^3-x}{x}, & x \ne 0 \\ -1, & x = 0 \end{cases}$$

hole @ (0,-1)

Which of the following statements are true for this function?

✓ *I.* Function f is continuous at $x = 0$.

✓ *II.* $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

✓ *III.* $\lim_{x \to 0} f(x) = f(0)$

a) *I* and *III*, only
b) *I* and *II*, only
c) *I*, *II*, and *III*
d) *II*, only
e) *III*, only

E 32) For what value of k is the function below continuous?

$$g(x)=\begin{cases} kx-2, & x \le -1 \\ kx^2+3, & x > -1 \end{cases}$$

$kx-2 = kx^2+3$ @ $x=-1$

$-k-2 = +k+3$

$-2x=5$

$x=-5/2$

a) $\frac{1}{2}$
b) -1
c) $\frac{5}{2}$
d) $-\frac{1}{2}$
e) $-\frac{5}{2}$

C 33) Function f is continuous for all real numbers, with f(-3) = -5 and f(1) = 2. If function f has *exactly one* zero, then its x value could be what number?

a) -4
b) -3
c) -1
d) 1
e) 4

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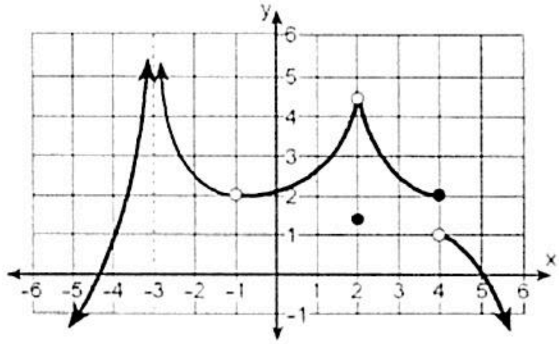
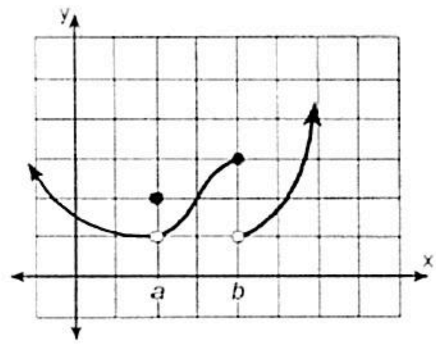
A 34) The graph of function f is shown below.



For what value(s) of x does the limit of f(x) fail to exist?

a) $x = -3$ and $x = 4$, only
~~b)~~ $x = -3$, $x = -1$, and $x = 4$, only
~~c)~~ $x = -1$, $x = 2$, and $x = 4$, only
~~d)~~ $x = -3$, only
e) $x = -3$, $x = -1$, $x = 2$, and $x = 4$

E 35) The graph of function f is shown below.



Which of the following statements about function f are true?

~~*I.*~~ $\lim_{x \to b} f(x)$ exists

✓ *II.* $\lim_{x \to a} f(x)$ exists

~~*III.*~~ $\lim_{x \to a} f(x) = f(a)$

a) *III*, only
b) *II* and *III*, only
c) *I*, *II*, and *III*
d) *I* and *II*, only
e) *II*, only