AP Calculus BC Name ______________________________

Chapter 5 Review (5.1-5.6)

AP Problems

1) $\int_1^2 \frac{1}{x^2}\,dx =$ $\int_1^2 x^{-2}dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$

(D) 1 E) $2\ln 2$

2) If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$ $\sqrt{x^3+1}$ @ $2 = \sqrt{2^3+1} = 3$

(A) – 3 (B) – 2 (C) 2

(D) 3 (E) 18

3) The average value of $\cos x$ on the interval [3, 5] is $\frac{1}{5-3}\int_3^5 \cos x\,dx = \frac{1}{2}\left[\sin x\Big|_3^5\right] = \frac{\sin 5 - \sin 3}{2}$

(A) $\frac{\sin 5 - \sin 3}{8}$ (B) $\frac{\sin 5 - \sin 3}{2}$ (C) $\frac{\sin 3 - \sin 5}{2}$

(D) $\frac{\sin 3 + \sin 5}{2}$ (E) $\frac{\sin 3 + \sin 5}{8}$

4) $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$ $\int \frac{\cancel{3x^2}}{u^{1/2}} \cdot \frac{du}{\cancel{3x^2}} = \int u^{-1/2}du = \frac{u^{1/2}}{\frac{1}{2}} = 2\sqrt{x^3+1} + C$

$u = x^3+1$
$du = 3x^2\,dx$
$dx = \frac{du}{3x^2}$

(A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$ (E) $\ln(x^3+1)+C$

5) $\int (x^2+1)^2\,dx =$ $\int (x^4+2x^2+1)\,dx = \frac{x^5}{5} + \frac{2}{3}x^3 + x + C$

(A) $\frac{(x^2+1)^3}{3}+C$ (B) $\frac{(x^2+1)^3}{6x}+C$ (C) $\left(\frac{x^3}{3}+x\right)^2+C$

(D) $\frac{2x(x^2+1)^3}{3}+C$ (E) $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

6) $\int_0^x \sin t\, dt =$ $-\cos t \Big|_0^x = -\cos x - (-\cos 0) = -\cos x + 1$

$= 1 - \cos x$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$
(D) $\cos x - 1$ (E) $1 - \cos x$

7) What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2]?

$\frac{1}{2}\int_0^2 x^2\sqrt{x^3+1}\,dx$ $u = x^3+1$, $du = 3x^2dx$, $dx = \frac{du}{3x^2}$

$\frac{1}{6}\int_0^2 u^{1/2}du$

$\frac{1}{6}\left[\frac{u^{3/2}}{3/2}\right. = \frac{1}{9}\left[(x^3+1)^{3/2}\right]_0^2$

$= \frac{1}{9}[27-1]$

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$
(D) $\frac{52}{3}$ (E) 24

8) $\int x^2\cos(x^3)\,dx =$ $\frac{1}{3}\int \cos u\, du = \frac{1}{3}\sin(x^3) + C$

$u = x^3$, $du = 3x^2dx$, $dx = \frac{du}{3x^2}$

(A) $-\frac{1}{3}\sin(x^3)+C$ (B) $\frac{1}{3}\sin(x^3)+C$ (C) $-\frac{x^3}{3}\sin(x^3)+C$
(D) $\frac{x^3}{3}\sin(x^3)+C$ (E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

9) Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

$u = 2x+1$, $du = 2dx$, $dx = \frac{du}{2}$

(A) $\frac{1}{2}\int_{-1/2}^{1/2}\sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2\sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5\sqrt{u}\,du$
(D) $\int_0^2\sqrt{u}\,du$ (E) $\int_1^5\sqrt{u}\,du$

$\frac{1}{2}\int_0^2\sqrt{u}\,du$

$0: 2(0)+1 = 1$
$2: 2(2)+1 = 5$
$\frac{1}{2}\int_1^5\sqrt{u}\,du$

10) $\int_0^8 \frac{dx}{\sqrt{1+x}} =$ $\int_0^8 u^{-1/2}du = 2u^{1/2} \to 2\sqrt{1+x}\Big|_0^8 = 6-2=4$

$u = 1+x$, $du = dx$

(A) 1 (B) $\frac{3}{2}$ (C) 2
(D) 4 (E) 6

11) $\int \sin(2x+3)\,dx =$ $\frac{1}{2}\int \sin u\,du =$ $\frac{1}{2}(-\cos u) =$

$u = 2x+3$
$du = 2dx$
$dx = \frac{du}{2}$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

12) For $0 \le t \le 3$, a particle is moving along the x-axis. The velocity of the particle is given by $v(t) = t^2 - 4$. (make up units if units aren't given)

a) Find the <u>average velocity</u> of the particle for the time period $0 \le t \le 3$.

$\frac{1}{3}\int_0^3 (t^2-4)\,dt = \frac{1}{3}\left[\frac{t^3}{3} - 4t\right]_0^3 = \frac{1}{3}((9-12)-0) = \boxed{-1 \text{ ft/sec}}$

b) How far from its starting position is the particle at time t = 3?

$s(3) - s(0) = -3$

$\boxed{\text{3 units to the left}}$

t (hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

13) A zoo sponsored a one-day contest to name a baby elephant. Zoo visitors deposited entries in a special box between noon (t = 0) and 8 P.M. (t = 8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(t), in hundreds of entries, at various times t are shown in the table above.

a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time t = 6. Show the computations that lead to your answer.

$\frac{E(7)-E(5)}{7-5} = \frac{21-13}{2} = 4$ $\boxed{\text{400 entries/hr}}$

b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)dt$ in terms of the number of entries.

$\frac{1}{8}\left(\frac{2}{2}(0+4) + \frac{3}{2}(4+13) + \frac{2}{2}(13+21) + \frac{1}{2}(21+23)\right) = \frac{1}{8}(88.5) = 10.6875$

1068.75 entries
Means avg # of entries submitted over 8 hrs.

t (hours)	0	2	5	7	8	10
v(t) (miles per hour)	50	55	60	70	65	75

14) The table above gives the velocity v(t) at selected times t of a car traveling along a straight road.

→ AROC of V(t)

a) Find the average acceleration of the car over the interval $2 \le t \le 8$. Show the work that leads to your answer and indicate units of measure.

$\frac{V(8)-V(2)}{8-2} = \frac{65-55}{6} = \frac{10}{6}$ $\boxed{5/3 \text{ miles/hr}}$

b) Use a left Riemann Sum with the subintervals given in the table to approximate $\int_0^{10} v(t)dt$. Indicate units of measure. LRAM: $2(50)+3(55)+2(60)+1(70)+2(65)$

$100+165+120+70+130$

$\boxed{585 \text{ miles}}$

c) What physical quantity does the integral in part b represent? $\boxed{\text{Total miles traveled from 0 to 10 hrs}}$

x	2	3	5	8	13
$f(x)$	1	4	−2	3	6

15) Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval [2, 13].

a) Estimate f'(4). Show the work that leads to your answer.

$f'(4) \approx \frac{f(5)-f(3)}{5-3} = \frac{-2-4}{2} = \boxed{-3}$

b) Evaluate $\int_2^{13} (3-5f'(x))dx$. Show the work that leads to your answer.

$\int_2^{13} 3dx - 5\int_2^{13} f'(x)dx = 3x\Big|_2^{13} - 5\left[f(x)\Big|_2^{13}\right] = (3(13)-3(2)) - 5(f(13)-f(2))$

$(39-6)-5(6-1) = 3-25$

$= \boxed{-22}$

c) Use a right Riemann sum with subintervals indicated by the date in the table to approximate $\int_2^{13} f(x)dx$. Show the work that leads to your answer.

RRAM: $5(6)+3(3)+2(-2)+1(4) =$

$30+9-4+4 = \boxed{39}$

Extra Practice

Evaluate each indefinite integral.

16) $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx$

$= \frac{x^{5/3}}{5/3} + C$

$= \boxed{\frac{3}{5}x^{5/3} + C}$

17) $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$

$= \frac{x^{-2}}{-2} + C$

$= \boxed{-\frac{1}{2x^2} + C}$

18) $\int \frac{x^2+x+1}{\sqrt{x}}\,dx = \int\left(\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}} + x^{-1/2}\right)dx$

$= \int\left(x^{3/2} + x^{1/2} + x^{-1/2}\right)dx$

$= \boxed{\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

19) $\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx =$

$\boxed{x^3 + \frac{x^2}{2} - 2x + C}$

20) $\int (2\sin x + 3\cos x)\,dx =$

$\boxed{-2\cos x + 3\sin x + C}$

21) $\int (1 - \csc x \cot x)\,dx = \int 1\,dx - \int \csc x \cot x\,dx$

$= \boxed{x + \csc(x) + C}$

22) $\int (\sec^2 x - \sin x)\,dx =$

$\boxed{\tan x + \cos x + C}$

23) $\int 3x^{-4}\,dx =$

$\frac{3x^{-3}}{-3} = \boxed{-\frac{1}{x^3} + C}$

24) $\int x(x^2-1)^4\,dx =$

$u = x^2 - 1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int x\,u^4 \frac{du}{2x} =$

$\frac{1}{2}\int u^4\,du =$

$\frac{1}{2}\frac{u^5}{5} = \boxed{\frac{(x^2-1)^5}{10} + C}$

25) $\int x\sqrt{x+1}\,dx =$

$u = x + 1$

$du = dx$

$x = u - 1$

$\int x(x+1)^{1/2}\,dx =$

$\int (u-1)u^{1/2}\,du =$

$\int (u^{3/2} - u^{1/2})\,du =$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + \ldots}$

26) $\int \sin^3 3x \ \cos 3x \ dx$

$u = \sin(3x)$
$du = 3\cos(3x)dx$
$dx = \frac{du}{3\cos(3x)}$

$\int u^3 \cancel{\cos(3x)} \frac{du}{3\cancel{\cos(3x)}} =$

$\frac{1}{3}\int u^3 du = \frac{1}{3}\frac{u^4}{4}$

$\boxed{\frac{1}{12}\sin^4(3x) + C}$

27) $\int x\sec^2 x^2 dx =$

$u = x^2$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\frac{1}{2}\int \sec^2 u \, du =$

$\boxed{\frac{1}{2}\tan(x^2) + C}$

28) $\int \frac{7 + 3x^{3/2}}{\sqrt{x}} dx = \int (7x^{-1/2} + 3x) dx$

$\boxed{14x^{1/2} + \frac{3}{2}x^2 + C}$

29) $\int x^2 (x^3 - 1)^4 dx$

$u = x^3 - 1$
$du = 3x^2 dx$
$dx = \frac{du}{3x^2}$

$\frac{1}{3}\int u^4 du$

$\frac{1}{3}\frac{u^5}{5} = \boxed{\frac{(x^3-1)^5}{15} + C}$

30) Find the <u>approximate</u> area of the region bounded by the graph of y = 3x – 4 and the x-axis between x = 2 and x = 5, using 6 equal width rectangles.

a) left sum

x	2	2.5	3	3.5	4	4.5	5
y	2	3.5	5	6.5	8	9.5	11

$.5(2 + 3.5 + 5 + 6.5 + 8 + 9.5)$

$= .5(34.5) = \boxed{17.25}$

b) right sum

$.5(11 + 9.5 + 8 + 6.5 + 5 + 3.5)$

$.5(43.5) = \boxed{21.75}$

Evaluate each definite integral.

31) $\int_{-1}^{0} \left(t^{1/3} - t^{2/3}\right) dt =$

$\left. \frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3} \right|_{-1}^{0}$

$0 - \left(\frac{3}{4} + \frac{3}{5}\right) =$

$\boxed{-\frac{27}{20}}$

32) $\int_{0}^{\pi} (1 + \sin x) dx =$

$\left. x - \cos x \right|_{0}^{\pi} =$

$(\pi - \cos\pi) - (0 - \cos 0) =$

$\pi + 1 + 1 = \boxed{\pi + 2}$

33) $\int_0^1 x\sqrt{x^2+1}\,dx =$

$u = x^2+1$
$du = 2x\,dx$
$dx = \frac{du}{2x}$
$0 :\to 1$
$1 :\to 2$

$\frac{1}{2}\int_0^1 u^{1/2}\,du =$

$\frac{1}{3}\left[u^{3/2}\right|_1^2 =$

$\frac{1}{3}\left[\sqrt{8}-1\right] = \boxed{\frac{2\sqrt{2}-1}{3}}$

34) $\int_{-1}^{1} x(x^2+1)^3\,dx$

$u = x^2+1$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\frac{1}{2}\int_{-1}^{1} u^3\,du$

$\frac{1}{2}\left[\frac{u^4}{4}\right] = \frac{1}{8}\left[(x^2+1)^4\right|_{-1}^{1} =$

$\frac{1}{8}\left[(2)^4-(2)^4\right] = \boxed{0}$

35) $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx =$

$u = 2x+1$
$du = 2dx$
$dx = \frac{du}{2}$

$\frac{1}{2}\int_0^4 u^{-1/2}\,du$

$u^{1/2} \to \sqrt{2x+1}\,\Big|_0^4 =$

$3-1 = \boxed{2}$

36) $\int_0^7 x\sqrt[3]{x+1}\,dx$

$u = x+1 \to x = u-1$
$du = dx$
$0 :\to 1$
$7 :\to 8$

$\int_1^8 (u-1)u^{1/3}\,du =$

$\int_1^8 (u^{4/3}-u^{1/3})\,du =$

$\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\Big|_1^8 = \left(\frac{3}{7}(128)-\frac{3}{4}(16)\right)-\left(\frac{3}{7}-\frac{3}{4}\right)$

$\approx \boxed{43.179}$

37) Determine the area of the region bounded by y = x – x² and the x-axis between x = 0 and x = 1.

$\int_0^1 (x-x^2)\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^1$

$= \left(\frac{1}{2}-\frac{1}{3}\right)-(0)$

$= \boxed{\frac{1}{6}}$

38) Find an equation for the function f that has the derivative $f'(x) = -2x\sqrt{8-x^2}$ and whose graph passes through the point (2, 7).

$u = 8-2x^2$
$du = -4x\,dx$
$dx = \frac{du}{-4x}$

$\int -2x\sqrt{8-x^2}\,dx = f(x)$

$\frac{1}{2}\int u^{1/2}\,du = f(x)$

$\frac{1}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = f(x)$

$\frac{1}{3}(8-2x^2)^{\frac{3}{2}} + C = f(x)$

$\frac{1}{3}(8-2(2)^2)^{3/2} + C = 7$

$0 + C = 7$

$\boxed{f(x) = \frac{1}{3}(8-2x^2)^{3/2} + 7}$

39) Find the average value of $f(x) = 2x^2 + 3$ on the interval [0, 2].

$$\frac{1}{2}\int_0^2 (2x^2+3)\,dx = \frac{1}{2}\left[\frac{2}{3}x^3 + 3x\Big|_0^2\right.$$
$$= \frac{1}{2}\left[\left(\frac{16}{3}+6\right)-(0)\right]$$
$$= \frac{1}{2}\left(\frac{34}{3}\right) = \boxed{34/6}$$

Find F'(x). FTC!!

40) $F(x) = \int_1^x (3t^2 - 4)\,dt$

$$\boxed{F'(x) = 3x^2 - 4}$$

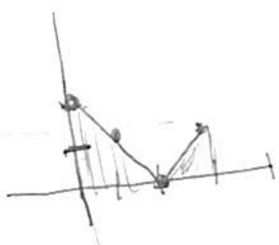
41) $F(x) = \int_4^{4x} t^4\,dt$

$$F'(x) = (4x)^4 \cdot 4$$
$$\boxed{F'(x) = 1024x^4}$$

42) Evaluate the integral: $\int_0^3 |x-2|\,dx$ area under "curve" from 0 to 3

since abs value, consider

$-(x-2) \le 0 \le x-2$



$$= \int_0^2 (-x+2)\,dx + \int_2^3 (x-2)\,dx$$
$$= -\frac{x^2}{2} + 2x\Big|_0^2 + \frac{x^2}{2} - 2x\Big|_2^3$$
$$-2+4 + \left[\left(\frac{9}{2}-6\right)-(2-4)\right]$$
$$2 + \frac{1}{2} = \boxed{2.5}$$

43) Approximate the value of $\int_1^2 \frac{1}{(x+1)^2}\,dx$ using 4 trapezoids.

$h = \Delta x = .25$

Put in y_1=

home scn

using VARS

$$\frac{.25}{2}\Big[(f(1)+f(1.25)) + (f(1.25)+f(1.5)) + (f(1.5)+f(1.75)) +$$
$$(f(1.75)+f(2))\Big)$$

(bases)

Inner y-values are used twice every time in trapezoid rule!

So..

$$\frac{.25}{2}\left(f(1) + 2f(1.25) + 2f(1.5) + 2(f1.75) + f(2)\right)$$

$$\approx \boxed{.168}$$ verify approx w/calc $\rightarrow$ = .166 .167 ☺