AP Calculus AB Name KEY

Review for Test 4.1, 4.3, 4.4

NO Calculator!

1) Locate the absolute extrema of the function $f(x) = 2x^2 - 4x - 2$ on the closed interval [-2, 2].

EVT

x	$f(x)$
-2	14
1	-4
2	-2

$f'(x) = 4x - 4$
$0 = 4x - 4$
$4 = 4x$
$x = 1$

$\boxed{f(x) \text{ absolute max @ } (-2, 14) \quad f(x) \text{ absolute min @ } (1, -4)}$

2) Locate the absolute extrema of the function $f(x) = x + \cos x$ on the closed interval [0, 2π].

EVT

x	$f(x)$
0	1
$\frac{\pi}{2}$	$\pi/2$
2π	$2\pi + 1$

$f'(x) = 1 - \sin x$
$0 = 1 - \sin x$
$\sin x = 1$
$x = \frac{\pi}{2}$

$\boxed{f(x) \text{ absolute min @ } (0, 1) \quad f(x) \text{ absolute max @ } (2\pi, 2\pi + 1)}$

3) For the function $f(x) = 2x^3 - 24x^2 + 5$:

a) Find the critical number(s) of the function.
b) Find the intervals where the function is increasing or decreasing.
c) Apply the First Derivative Test to identify all relative extrema.
d) Find the "possible" points of inflection of the function.
e) Find the intervals where the function is concave up or concave down.
f) Find the points of inflection of the function.

a) $f'(x) = 6x^2 - 48x$
$0 = 6x(x - 8)$

$\boxed{\text{Critical \#s of } f(x) \quad x = 0 \quad x = 8}$

b) + 0 − 8 +

$\boxed{\begin{array}{l} f(x) \text{ increasing from } (-\infty, 0) \cup (8, \infty) \\ f(x) \text{ decreasing from } (0, 8) \end{array}}$

c) $\boxed{\begin{array}{l} f(x) \text{ has relative max @ } (0, 5) \\ \quad \text{b/c } f'(x) \text{ changes from + to −} \\ f(x) \text{ has a relative min @ } (8, -507) \\ \quad \text{b/c } f'(x) \text{ changes from − to +} \end{array}}$

d) $f''(x) = 12x - 48$
$0 = 12(x - 4)$
$x = 4$

$\boxed{f(x) \text{ PPOI @ } x = 4}$

e) − 4 +

$\boxed{\begin{array}{l} f(x) \text{ is concave down from } (-\infty, 4) \\ f(x) \text{ is concave up from } (4, \infty) \end{array}}$

f) $\boxed{\begin{array}{l} \text{POI of } f(x) \text{ @ } (4, -251) \\ \quad \text{b/c } f''(x) \text{ changes from − to +} \end{array}}$

4) Identify the open intervals where the function $f(x) = 3x^4 + x^3 - 21x^2$ is increasing or decreasing.

$f'(x) = 12x^3 + 3x^2 - 42x$

$f'(x) = 3x(4x^2 + x - 14)$

$f'(x) = 3x(4x - 7)(x + 2)$

$0 = 3x(4x - 7)(x + 2)$

$x = 0 \quad x = 7/4 \quad x = -2$

$- \quad -2 \quad + \quad 0 \quad - \quad 7/4 \quad +$

$\boxed{\begin{array}{l} f(x) \text{ is increasing from } (-2,0) \cup (7/4,\infty) \\ f(x) \text{ is decreasing from } (-\infty,-2) \cup (0,7/4) \end{array}}$

5) Find all relative extrema: $y = \dfrac{2x}{(x+4)^3}$

$\frac{dy}{dx} = \frac{(x+4)^3(2) - 2x(3)(x+4)^2(1)}{(x+4)^6}$

$\frac{dy}{dx} = \frac{2(x+4)^2(x+4-3x)}{(x+4)^6}$

$\frac{dy}{dx} = \frac{2(-2x+4)}{(x+4)^4}$

$\frac{dy}{dx} = \frac{-4(x-2)}{(x+4)^4}$

Critical #s: $x = 2, -4$

$\frac{dy}{dx}$

$+ \quad -4 \quad + \quad 2 \quad -$

$\boxed{\begin{array}{l} y \text{ has a relative max @ } (2, \frac{1}{54}) \\ \text{b/c } \frac{dy}{dx} \text{ changes from + to} - \end{array}}$

6) Determine the open intervals on which the graph of $f(x) = 7 - 15x + 9x^2 - x^3$ is concave downward or concave upward.

$f'(x) = -15 + 18x - 3x^2$

$f''(x) = 18 - 6x$

$0 = 18 - 6x$

$6x = 18$

PPOI $x = 3$

$f''(x)$

$+ \quad 3 \quad -$

$\boxed{\begin{array}{l} f(x) \text{ is concave up from } (-\infty,3) \\ f(x) \text{ is concave down from } (3,\infty) \end{array}}$

7) Find all points of inflection: $f(x) = \dfrac{1}{12}x^4 - 2x^2 + 15$.

$f'(x) = \frac{1}{3}x^3 - 4x$

$f''(x) = x^2 - 4$

$0 = (x+2)(x-2)$

PPOI $x = -2; 2$

$+ \quad -2 \quad - \quad 2 \quad +$

$f(-2) = \frac{4}{3} - \frac{24}{3} + \frac{45}{3}$

$f(2) = \frac{4}{3} - \frac{24}{3} + \frac{45}{3}$

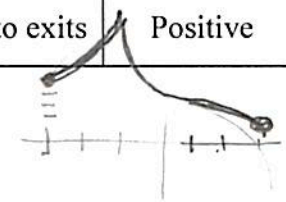
$\boxed{\begin{array}{l} f(x) \text{ has POI @ } (-2, \frac{25}{3}) \text{ b/c } f''(x) \text{ changes from + to} - \\ f(x) \text{ has a POI @ } (2, \frac{25}{3}) \text{ b/c } f''(x) \text{ changes from} - \text{to +} \end{array}}$

8) <u>1984 AB 4 and BC 3</u>

A function f is continuous on the closed interval $[-3,3]$ such that $f(-3)=4$ and $f(3)=1$. The function f' and f'' have the properties given in the table below.

x	$f(x)$
-3	4
3	1

x	$-3<x<-1$	$x=-1$	$-1<x<1$	$x=1$	$1<x<3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exits	Positive	0	Negative



(a) What are the x-coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3,3]$? Justify your answer.

f has an absolute maximum @ X=-1 because f'(x) changes from + to - @ X=-1 and continues to decrease after X=1

f has an absolute minimum @ X=3 because it must occure at an end point or X=1; because it decreases after X=1 the lowest point is at X=3

(b) What are the x-coordinates of all points of inflection of f on the interval $[-3,3]$? Justify your answer.

f has a point of inflection @ X=1 only because it is the only place on the graph where the second derivative changes signs. (+ to −)

(c) Sketch a graph that satisfies the given properties of f.