AP Calculus AB

$\frac{d}{dx} a^u = \ln a\,(a^u)(u')$

$\frac{d}{dx} \log_a u = \frac{u'}{\ln a\,(u)}$

Name KEY

Review for Test 3.1 – 3.5, 3.7

Find the derivative of each function.

1) $y = \dfrac{x^3 - 2x + 5}{x^2 + 4}$

$3x^4 - \cancel{2x^2} + 12x^2 - 8 - \cancel{2x^4} + \cancel{4x^2} - 10x$

$\frac{dy}{dx} = \dfrac{(x^2+4)(3x^2-2) - (x^3-2x+5)(2x)}{(x^2+4)^2}$

$\boxed{\frac{dy}{dx} = \dfrac{x^4 + 14x^2 - 10x - 8}{(x^2+4)^2}}$

2) $f(x) = (3x^2 - 2x)^{4/3}$

$f'(x) = \frac{4}{3}(3x^2-2x)^{\frac{1}{3}}(6x-2)$

$\boxed{f'(x) = (8x - \frac{8}{3})(3x^2-2x)^{\frac{1}{3}}}$

3) $y = \cos^4(3x^2)$ try "u" substitution

$u = \cos(3x^2)$ $\quad u' = -6x\sin(3x^2)$

$y = u^4$

$\frac{dy}{du} = 4u^3 u'$ $\quad \frac{dy}{dx} = 4(\cos(3x^2))^3(-6x\sin(3x^2))$

$\boxed{\frac{dy}{dx} = -24x\cos^3(3x^2)\sin(3x^2)}$

4) $y = x^2\sqrt{1-x^2}$ $\quad y = x^2(1-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = (1-x^2)^{\frac{1}{2}}(2x) + x^2\left(\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\right)(-2x)$

$\boxed{\frac{dy}{dx} = 2x(1-x^2)^{\frac{1}{2}} - \dfrac{x^3}{(1-x^2)^{\frac{1}{2}}}}$ S

or

$\boxed{\frac{dy}{dx} = \dfrac{-3x^3 + 2x}{\sqrt{1-x^2}}}$

5) f(x) = (3 – 2x + x³)(x⁴ + 7)

$f(x) = 3x^4 + 21 - 2x^5 - 14x + x^7 + 7x^3$

$f(x) = x^7 - 2x^5 + 3x^4 + 7x^3 - 14x + 21$

$\boxed{f'(x) = 7x^6 - 10x^4 + 12x^3 + 21x^2 - 14}$

6) $f(x) = x^2\sin 2x$

$f'(x) = \sin(2x)(2x) + x^2\cos(2x)(2)$

$\boxed{f'(x) = 2x\sin(2x) + 2x^2\cos(2x)}$

7) $g(x) = \ln\sqrt{x}$

$g(x) = \ln x^{\frac{1}{2}}$

$g(x) = \frac{1}{2}\ln x$

$\boxed{g'(x) = \dfrac{1}{2x}}$

8) $f(x) = x\sqrt{\ln x}$ not the same as $\ln\sqrt{x}$

$f(x) = x(\ln x)^{\frac{1}{2}}$

$f'(x) = (\ln x)^{\frac{1}{2}}(1) + \cancel{x}\left(\frac{1}{2}(\ln x)^{-\frac{1}{2}}\right)\left(\frac{1}{\cancel{x}}\right)$

$\boxed{f'(x) = (\ln x)^{\frac{1}{2}} + \dfrac{1}{2(\ln x)^{\frac{1}{2}}} \text{ or } \dfrac{2\ln(x) + 1}{2\sqrt{\ln x}}}$

9) $h(x) = \ln\dfrac{x(x-1)}{x-2}$

$h(x) = \ln x + \ln(x-1) - \ln(x-2)$

$\boxed{h'(x) = \dfrac{1}{x} + \dfrac{1}{(x-1)} - \dfrac{1}{(x-2)}}$ S

$x^2 - 3x + 2 + \cancel{x^2} - 2x - \cancel{x^2} + x$

$h'(x) = \dfrac{(x-1)(x-2) + x(x-2) - x(x-1)}{x(x-1)(x-2)}$

$h'(x) = \dfrac{x^2 - 4x + 2}{x(x-1)(x-2)}$

10) $f(x) = \ln|\sin x|$

$\frac{d}{dx}\ln|u| = \dfrac{u'}{\ln e\, u}$

$f'(x) = \dfrac{\cos x}{\sin x}$

$\boxed{f'(x) = \cot x}$

11) $g(t) = t^2e^t$

$g'(t) = e^t(2t) + t^2e^t$

$\boxed{g'(x) = e^t(t^2+2t) \text{ or } te^t(t+2)}$

12) $y = \sqrt{e^{2x} + e^{-2x}}$

$y = (e^{2x}+e^{-2x})^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(e^{2x}+e^{-2x})^{\frac{-1}{2}}(2e^{2x} - 2e^{-2x})$

$\boxed{\frac{dy}{dx} = \frac{\left(2e^{2x} - \frac{2}{e^{2x}}\right)}{2\sqrt{e^{2x} + \frac{1}{e^{2x}}}}}$

13) $g(x) = \frac{x^2}{e^x}$

$g'(x) = \frac{e^x(2x) - x^2(e^x)}{e^{2x}}$

$g'(x) = -\frac{x\cancel{e^x}(x-2)}{e^{\cancel{2}x}}$ $\boxed{g'(x) = \frac{-x(x-2)}{e^x}}$

14) $g(x) = \ln\frac{e^x}{1+e^x}$

$g(x) = \ln e^x - \ln(1+e^x)$

$g(x) = x\ln e - \ln(1+e^x)$

$g(x) = x - \ln(1+e^x)$

$\boxed{g'(x) = 1 - \frac{e^x}{1+e^x}}$

15) $f(x) = e^{\cos x}$

$\boxed{f'(x) = -\sin x\,(e^{\cos x})}$

16) $g(x) = \log_3\sqrt{1-x}$

$g(x) = \frac{1}{2}\log_3(1-x)$

$g'(x) = \frac{1}{2}\left(\frac{-1}{\ln(3)(1-x)}\right)$

$\boxed{g'(x) = \frac{-1}{2(\ln 3)(1-x)}}$

17) $y = x(4^{-x})$

$\frac{dy}{dx} = 4^{-x}(1) + x(\ln 4)(4^{-x})(-1)$

$\frac{dy}{dx} = \frac{1}{4^x} + \frac{-x(\ln 4)}{4^x}$

$\boxed{\frac{dy}{dx} = \frac{1 - x\ln 4}{4^x}}$

18) $h(x) = \log_5\frac{x}{x-1}$

$h(x) = \log_5 x - \log_5(x-1)$

$\boxed{h'(x) = \frac{1}{(\ln 5)(x)} - \frac{1}{(\ln 5)(x-1)} \text{ or } \frac{-1}{(\ln 5)(x)(x-1)}}$

19) Given $f(x) = \sin^2 2x$, find f'(x) and then find $f'\left(\frac{\pi}{6}\right)$.

$f'(x) = 2(\sin(2x))(\cos(2x))(2)$

$f'(x) = 4\sin(2x)\cos(2x)$

$f'\left(\frac{\pi}{6}\right) = 4\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$

$\boxed{f'\left(\frac{\pi}{6}\right) = 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \sqrt{3}}$

20) Given f(x) = $(2x^3 - 4x - 1)^{4/3}$, find f'(x) and then find f'(-1).

$f'(x) = \frac{4}{3}(2x^3-4x-1)^{\frac{1}{3}}(6x^2-4)$

$f'(-1) = \frac{4}{3}(-2+4-1)(6-4)$

$\boxed{f'(-1) = \frac{8}{3}}$

21) Find an equation of the tangent line to the graph of $y = \ln(2+x) + \dfrac{2}{2+x}$ at the point (-1, 2).

$$\boxed{\begin{gathered} y-2=-1(x+1) \\ \text{or} \\ y=-x+1 \end{gathered}}$$

$$\frac{dy}{dx} = \frac{1}{x+2} + \frac{(x+2)(0)-2(1)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{(x+2)}{(x+2)^2} - \frac{2}{(x+2)^2} = \frac{x}{(x+2)^2} \quad @ \quad \frac{-1}{(1)^2} = -1$$

22) Find an equation of the tangent line to the graph of $f(x) = \ln(e^{-x^2})$ at the point (2, -4).

$$\boxed{\begin{gathered} y+4=-4(x-2) \\ \text{or} \\ y=-4x+4 \end{gathered}}$$

$$f'(x) = \frac{-2x(e^{-x^2})}{e^{-x^2}} \qquad u' = e^{-x^2}(-2x)$$

$$f'(x) = -2x \quad @ \quad (2,4) \qquad f'(2) = -4$$

23) Find an equation of the tangent line to the graph of $y = \log_3 x$ at the point (81, 4).

$$\boxed{y-4 = \frac{1}{81\ln(3)}(x-81)}$$

$$\frac{dy}{dx} = \frac{1}{\ln(3)\,x} \quad @ \quad (81,4)$$

$$\frac{dy}{dx} = \frac{1}{81\ln 3}$$

24) Find the second derivative of y = $5x^3 - 7x^2 + 2x$.

$$y' = 15x^2 - 14x + 2$$

$$\boxed{y'' = 30x - 14}$$

25) Find the second derivative of f(x) = sin x + cos x.

$$f'(x) = \cos x - \sin x$$

$$\boxed{f''(x) = -\sin x - \cos x}$$

26) Two functions, f and g, are continuous and differentiable for all real numbers. Values of the functions and their derivatives are shown in the table.

x	-3
f(x)	2
f'(x)	6
g(x)	-3
g'(x)	4

a) If h(x) = 2f(x) – 5g(x), find h'(x) and then find h'(-3).

$h'(x) = 2f'(x) - 5g'(x)$

$h'(-3) = 2(6) - 5(4)$

$\boxed{h'(-3) = -8}$

b) If h(x) = $(f(x))^2$, find h'(x) and then find h'(-3).

$h'(x) = 2(f(x))f'(x)$

$h'(-3) = 2(2)(6)$

$\boxed{h'(-3) = 24}$

c) If h(x) = $2f(x) + (g(x))^2$, find h'(x) and then find h'(-3).

$h'(x) = 2f'(x) + 2(g(x))g'(x)$

$h'(-3) = 2(6) + 2(-3)(4)$

$\boxed{h'(-3) = -12}$

d) If h(x) = $f(g(x))$, find h'(x) and then find h'(-3).

$h'(x) = f'(g(x))g'(x)$

$h'(-3) = f'(-3)(4)$

$(6)(4)$

$\boxed{h'(-3) = 24}$

27) Find an equation of the tangent line to the graph of $f(x) = \dfrac{x-1}{2x+1}$ at the point (-1, 2).

$\boxed{\begin{array}{c} y-2 = 3(x+1) \\ \text{or} \\ y = 3x+5 \end{array}}$

$f'(x) = \dfrac{(2x+1)(1) - (x-1)(2)}{(2x+1)^2}$ @ $(-1,2)$ $\dfrac{-1+4}{1} = 3$

28) Given $f'(x) = \lim_{h \to 0} \dfrac{[3(x+h)^4 - 4(x+h)^2 + (x+h)] - [3x^4 - 4x^2 + x]}{h}$. (For the test, you need to be able to recognize this formula, find the f(x) and then find f'(x) on your own, they will not be specifically asked for on the test...you have to see the formula and know what to do! This formula was learned in section 3.1)

a) Find f(x). $3x^4 - 4x^2 + x$

b) Find f'(x). $12x^3 - 8x + 1$

c) Find f'(2). $96 - 16 + 1 = 81$

29) What is $\lim_{h \to 0} \dfrac{(x+h)^5 - x^5}{h}$. asking for derivative of $f(x) = x^5$

$\boxed{\lim_{h \to 0} \dfrac{(x+h)^5 - x^5}{h} = 5x^4}$

not instantaneous → No derivative

30) What is the average rate of change of y with respect to x from x = 2 to x = 5 when $y = x^2 - 3x$?

$$\frac{10-(-2)}{5-2} = \frac{12}{3} = \boxed{4}$$

$(2,-2)$ $(5,10)$

31) Given $x^2 - 2xy + y^3 = 10$, find $\frac{dy}{dx}$.

$$2x - 2\left(y(1) + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

$$2x - 2y - 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(-2x + 3y^2) = 2y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{2y-2x}{3y^2-2x}}$$

32) Find the slope of the tangent line to the graph of $3x^2 - y^3 = 11$ at the point (1, -2).

$$6x - 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6x}{-3y^2} = \frac{2x}{y^2} \quad @ (1,-2) \quad \frac{2}{4} = \frac{1}{2}$$

$$\boxed{\begin{array}{c} y + 2 = \frac{1}{2}(x-1) \\ \text{or} \\ y = \frac{1}{2}x - \frac{5}{2} \end{array}}$$

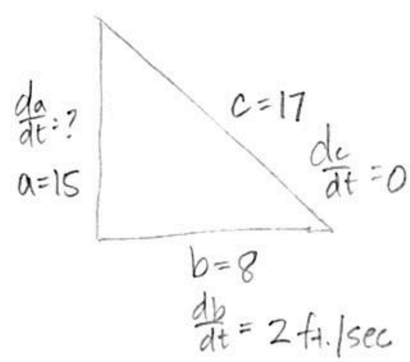
33) An extension ladder, 17 feet in length, is leaning against the side of a vertical wall. If the foot of the ladder is sliding away from the wall at a rate of 2 feet per second, how fast is the top of the ladder dropping when the foot is 8 feet from the base of the wall?

F: $\frac{da}{dt}$

W: $b = 8$

G: $\frac{db}{dt} = 2$

E: $a^2 + b^2 = c^2$



$8^2 + a^2 = 17^2$

$a = 15$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

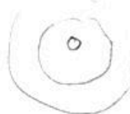
$$2(15)\frac{da}{dt} + 2(8)(2) = 2(17)(0)$$

$$30\frac{da}{dt} = -32$$

$$\boxed{\frac{da}{dt} = \frac{-16}{15} \text{ ft.sec}}$$ moving down@

34) The circular ripple caused by dropping a stone in a pond is increasing in area at a constant rate of 20 square meters per second. Determine how fast the radius of this circular ripple is increasing when the area of the circular region is 25π square meters.

F: $\frac{dr}{dt}$



$25\pi = \pi r^2$

$r=5$

W: $A = 25\pi$; $r=5$

G: $\frac{dA}{dt} = 20\ m^2/s$

E: $A=\pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$20 = 2\pi (5) \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{2}{\pi}\ m/sec.}$ radius getting bigger @

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

35) The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of v(t) are given in the table above.

a) What is the velocity of the particle at t = 20 seconds?

-10 m/s

b) Is the particle moving right or left at t = 32 seconds?

left... negative velocity

c) Find the <u>average acceleration</u> of the particle from t = 0 to t = 8 seconds.

Slope of velocity

$\frac{5-3}{8-0} = \frac{2}{8}$ or

$\boxed{\frac{1}{4}\ m/s^2}$ average acceleration