AP Calculus AB Name ~~K E Y~~

Review for Test 3.1 – 3.4

1) Use the definition of a derivative (limit process) to calculate the derivative of f(x) = x^2 + x.

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$ $f'(x) = \lim_{h \to 0} 2x + h + 1$

$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$ $\boxed{f'(x) = 2x + 1}$

$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$

2) Given $f'(x) = \lim_{h \to 0} \frac{[5(x+h)^4 - 8(x+h)^2 + 3] - [5x^4 - 8x^2 + 3]}{h}$.

a) What is f(x)? $\boxed{f(x) = 5x^4 - 8x^2 + 3}$ b) What is f'(x)? $\boxed{f'(x) = 20x^3 - 16x}$

3) If f(1) = 4 and f'(1) = 2, find an equation of the tangent line when x = 1.

(1, 4) m = 2

$\boxed{y - 4 = 2(x-1)}$ or $\boxed{y = 2x + 2}$

4) Find an equation of the tangent line to the graph of $f(x) = \frac{17}{x-1}$ at the point (2, 17).

$f(x) = 17(x-1)^{-1}$

$f'(x) = -17(x-1)^{-2}(1)$

$f'(x) = \frac{-17}{(x-1)^2}$ $f'(2) = \frac{-17}{1} = -17$

$\boxed{y - 17 = -17(x-2)}$

or

$\boxed{y = -17x + 51}$

5) Find an equation for the tangent line to the graph of f(x) = $-2x^2$ + 2x + 3 at the point where x = 1.

$f'(x) = -4x + 2$ $f(1) = 3$

$f'(1) = -4 + 2$

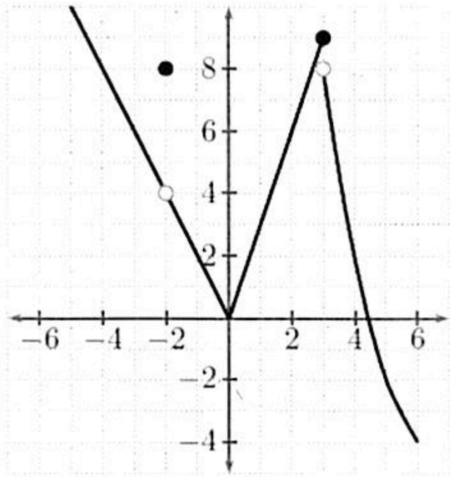
$f'(1) = -2$

$\boxed{y - 3 = -2(x-1)}$

or

$\boxed{y = -2x + 5}$

6) Below is the graph of a function f. Use the graph to answer the following:



a) Find all values of x where f is not continuous.

f is not continuous @ $x = -2; 3$

b) Find all values of x where f is not differentiable.

f is not differentiable @ $x = -2, 0, 3$

c) Find all values of x where f is continuous but not differentiable.

f is continuous but not differentiable @ $x = 0$

7) Consider $f(x) = \sqrt{x}$.

a) Find the derivative of f(x).
b) Find the slope of the tangent line to the graph of f at the point (4, 2).
c) Write an equation of the tangent line in part b.

a) $f(x) = x^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ or $\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$

b) $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

c) $\boxed{y - 2 = \frac{1}{4}(x-4)}$ or $\boxed{y = \frac{1}{4}x + 1}$

8) Find f'(x) if f(x) = $4x^4 - 5x^3 + 2x - 3$.

$\boxed{f'(x) = 16x^3 - 15x^2 + 2}$

9) Find f'(x) if $f(x) = \sqrt[3]{x^4} + \sqrt[4]{x^3}$.

$f(x) = x^{4/3} + x^{\frac{3}{4}}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{3}{4}x^{-\frac{1}{4}}$

Common den.

$f'(x) = \frac{(4x^{\frac{1}{4}})\,4x^{\frac{1}{3}}}{(4x^{\frac{1}{4}})\,3} + \frac{3\ (3)}{4x^{1/4}\ (3)}$

$f'(x) = \frac{16x^{\frac{7}{12}} + 9}{12x^{\frac{1}{4}}}$

$\boxed{f'(x) = \frac{16\sqrt[12]{x^7} + 9}{12\sqrt[4]{x}}}$

10) Find f'(x) if $f(x) = \frac{x^2 - 4x}{\sqrt{x}}$.

$f'(x) = \frac{x^{\frac{1}{2}}(2x-4) - (x^2-4x)\frac{1}{2}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}})^2}$

$f'(x) = \frac{2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}}{x}$

$f'(x) = \frac{\frac{3}{2}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}}{x} = \frac{3x^{\frac{1}{2}}}{2} - \frac{2}{x^{\frac{1}{2}}} = \boxed{\frac{3x-4}{2x^{\frac{1}{2}}} = f'(x)}$

11) Find the derivative of $y = \frac{1}{x^9}$.

$y = x^{-9}$

$y' = \boxed{\frac{-9}{x^{10}}}$

12) Find the <u>instantaneous rate of change</u> of w with respect to z if $w = \frac{7}{3z^2}$.

$W = \frac{7}{3} z^{-2}$

$w' = -\frac{14}{3} z^{-3}$

$\boxed{w' = \frac{-14}{3z^3}}$

13) Find all points at which the graph of f(x) = $x^3 - 3x$ has horizontal tangent lines.

$f'(x) = 0$

$f'(x) = 3x^2 - 3$

or

$f'(x) = 3(x^2 - 1)$

or

$f'(x) = 3(x-1)(x+1)$

$0 = 3(x-1)(x+1)$

f(x) has horizontal tang. lines @ (1, -2) and (-1, 2)

14) Find the <u>average rate of change</u> of y on the interval [1, 4], where y = $x^2 + x + 1$.

$\frac{y(4) - y(1)}{4-1} = \frac{21-3}{3} = \boxed{6}$

15) A coin is dropped from a height of 750 feet. The height, s (measured in feet) at time, t (measured in seconds), is given by s = $-16t^2 + 750$.

gravity constant

$V_0 = 0$ b/c dropped so no "x" term

a) Find the <u>average</u> velocity on the interval [1, 3].
b) Find the velocity function.
c) Find the instantaneous velocity when t = 3.
d) How long does it take for the coin to hit the ground?
e) Find the velocity of the coin when it hits the ground.
f) Find the acceleration function.
g) What is the acceleration of the coin when t = 4?

a) $\frac{s(3) - s(1)}{3-1} = \frac{606 - 734}{2} = \boxed{-64 \text{ ft/sec}}$

b) $\boxed{v(t) = -32t}$

c) $v(3) = \boxed{-96 \text{ ft/sec}}$

d) $0 = -16t^2 + 750$

$16t^2 = 750$

$t^2 = 46.875$

$t = \pm 6.85$

So after 6.846 seconds

e) $v(6.85) = \boxed{-219.09 \text{ ft/sec}}$

f) $\boxed{a(t) = v'(t) = -32}$

g) $\boxed{-32 \text{ ft/s}^2}$

16) Differentiate: $f(x)=\dfrac{x^2-1}{x^2+1}$.

$\cancel{2x^3}+2x-\cancel{2x^3}+2x$

$$f'(x)=\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}$$

$$\boxed{f'(x)=\frac{4x}{(x^2+1)^2}}$$

17) Find the derivative of $f(x)=(x^3+5x)(2x-7)$

$$f'(x)=(2x-7)(3x^2+5)+(x^3+5x)(2)$$

$$f'(x)=6x^3+10x-21x^2-35+2x^3+10x$$

$$\boxed{f'(x)=8x^3-21x^2+20x-35}$$

18) Find an equation of the tangent line to the graph of $f(x)=\dfrac{x-1}{x+1}$ when x = 1.

$$f'(x)=\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$$

$f(1)=0$

$$f'(x)=\frac{2}{(x+1)^2}$$

$$\boxed{y-0=\tfrac{1}{2}(x-1)}$$

$$f'(1)=\frac{2}{4}=\frac{1}{2}$$

19) Find the derivative of $y=9x^2\cdot\sin x$.

$$y'=\sin x\,(18x)+(9x^2)(\cos x)$$

$$\boxed{y'=9x\,(2\sin x+x\cos x)}$$

20) Differentiate: $f(x)=3(4x^3-9)^5$.

$$f'(x)=15(4x^3-9)^4(12x^2)$$

$$\boxed{f'(x)=180x^2(4x^3-9)^4}$$

21) Find the derivative of y = -x + tan x.

$$\boxed{y'=-1+\sec^2 x}$$

22) Find the derivative of $y=(x-2)^2\cdot\cos x$

$$y'=2(x-2)(1)(\cos x)+-\sin x\,(x-2)$$

$$\boxed{y'=(x-2)\left[2\cos x-(x-2)\sin x\right]}$$

23) Differentiate: $f(x)=\left(3x^2-2x+7\right)^{\frac{3}{4}}$

$$f'(x)=\frac{3}{4}(3x^2-2x+7)^{-\frac{1}{4}}(6x-2)$$

$$\boxed{f'(x)=\frac{3(6x-2)}{4(3x^2-2x+7)^{\frac{1}{4}}}}$$

24) Differentiate: $f(x)=\sin(3x^2-5)$.

$$f'(x)=6x\cos(3x^2-5)$$

25) Differentiate: $f(x)=5e^x+2\sin x-\sqrt{x}$

$f'(x)=5e^x+2\cos x-\frac{1}{2}x^{-\frac{1}{2}}$

$f'(x)=5e^x+2\cos x-\frac{1}{2x^{\frac{1}{2}}}$

or

$$\boxed{f'(x)=\frac{10e^x x^{\frac{1}{2}}+4x^{\frac{1}{2}}\cos x-1}{2x^{\frac{1}{2}}}}$$

26) Find $\frac{d}{dx}\left[e^x\sec 3x\right]$

$\frac{dy}{dx}=\sec 3x(e^x)+(e^x)\sec 3x\tan 3x(3$

$$\boxed{\frac{dy}{dx}=e^x\sec(3x)\left[3\tan(3x)+1\right]}$$

27) Differentiate: $f(x)=\cos^3(5x^2-7)$.

$u=\cos(5x^2-7)$

$u'=-\sin(5x^2-7)(10x)$

$f(u)=u^3$

$f'(u)=3u^2u'$

$f'(x)=3\left(\cos^2(5x^2-7)\right)\left(-\sin(5x^2-7)(10x)\right)$

$$\boxed{f'(x)=-30x\cos^2(5x^2-7)\sin(5x^2-7)}$$

28) Differentiate: $y=x^3\cdot\sqrt{4x-3}$

$y=x^3(4x-3)^{\frac{1}{2}}$

$y'=(4x-3)^{\frac{1}{2}}(3x^2)-(x^3)\frac{1}{2}(4x-3)^{-\frac{1}{2}}($

Common den.

$y'=\frac{(4x-3)^{\frac{1}{2}}\,3x^2(4x-3)^{\frac{1}{2}}}{(4x-3)^{\frac{1}{2}}}+\frac{2x^3}{(4x-3)^{\frac{1}{2}}}$

or

$y'=\frac{3x^2(4x-3)+2x^3}{(4x-3)^{\frac{1}{2}}}$

$$\boxed{y'=\frac{x^2(14x-9)}{(4x-3)^{\frac{1}{2}}}}$$

29) Find the value of f'(2) given that $f(x)=2\sqrt{4x^2-5x+9}$.

$$\boxed{f'(2)=\frac{11}{\sqrt{15}} \text{ or } \frac{11\sqrt{15}}{15}}$$

$f(x)=2(4x^2-5x+9)^{\frac{1}{2}}$

$f'(x)=(4x^2-5x+9)^{-\frac{1}{2}}(8x-5)$

$f'(x)=\frac{(8x-5)}{(4x^2-5x+9)^{\frac{1}{2}}}$

$f'(2)=\frac{16-5}{(16-10+9)^{\frac{1}{2}}}=\frac{11}{\sqrt{15}}$

30) Find the slope of $f(x)=\sin^2 x$ at the point where $x=\frac{3\pi}{4}$.

$f'(x)=2(\sin x)(\cos x)$

$f'\left(\frac{3\pi}{4}\right)=2\left(\sin\frac{3\pi}{4}\right)\left(\cos\frac{3\pi}{4}\right)$

$f'\left(\frac{3\pi}{4}\right)=2\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)=\boxed{-1}$

31) Find the second derivative of $f(x)=3(4x-5)^3$

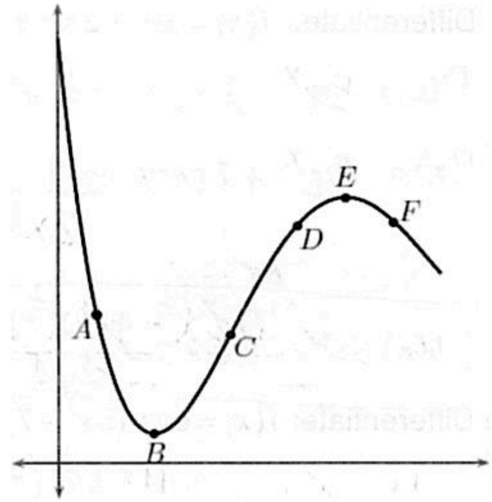
$f'(x)=9(4x-5)^2(4)$

$f''(x)=72(4x-5)(4)$

$$\boxed{\begin{aligned}f''(x)&=288(4x-5)\\ f''(x)&=1152x-1440\end{aligned}}$$

32) At which point(s) on the graph does the slope equal zero?

B & E

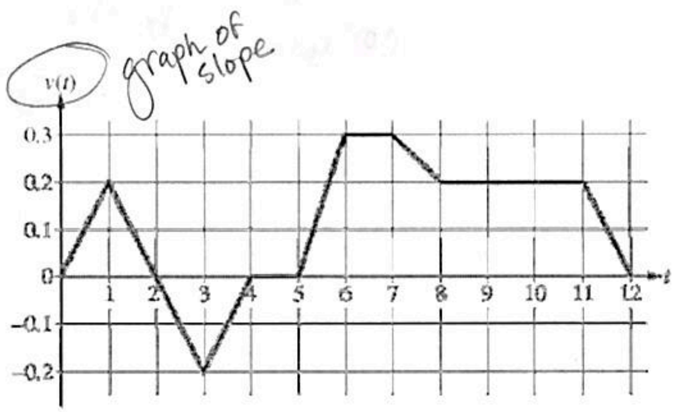


33) At which point on the graph does the slope change from positive to negative?

E

34) At which point on the graph does the slope change from negative to positive?

B



Caren rides a bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity v(t), in miles per minute, is modeled by the piecewise – linear function whose graph is shown above.

35) During what time interval(s) is Caren going towards school? (positive velocity)

$(0,2) \cup (5,12)$ or $0 < t < 2$ min. and $5 < t < 12$ min

36) During what time interval(s) is Caren riding back toward home? (negative slope)

$(2,4)$ $\boxed{2 < t < 4 \text{ min}}$

37) At what time(s) is Caren stopped? (no velocity)

$(4,5)$ $\boxed{4 < t < 5 \text{ min}}$

38) What is the acceleration of Caren's bicycle at time t = 6.5 minutes?

(slope of v curve)

Slope is $\boxed{0}$ so not accelerating or decelerating