Skip #47

HW
D3&4

Derivative Practice 3.2 - 3.5.

1. $y = (3x^4+5)^3$
$y' = 3(3x^4+5)^2(12x^3)$
$y' = 36x^3(3x^4+5)^2$

2. $y = \sec(\frac{1}{5}x)$
$y' = \frac{\sec(\frac{1}{5}x)\tan(\frac{1}{5}x)}{5}$

3. $f(x) = \frac{-2}{(3-x^2)^3}$
$f'(x) = \frac{-(-2)(3)(3-x^2)^2(-2x)}{(3-x^2)^6}$
$= \frac{-12x(3-x^2)^2}{(3-x^2)^6} = \frac{-12x}{(3-x^2)^4}$

4. $f'(x) = \frac{3(4x^2-7)^3 - [3x(3)(4x^2-7)^2]}{(4x^2-7)^6}$
$= \frac{3(4x^2-7)^3 - 9x(4x^2-7)^2}{(4x^2-7)^6}$
$= \frac{3(4x^2-7) - 9x}{(4x^2-7)^4}$

5. $y = (6x-2)^{1/2}$
$y' = \frac{1}{2}(6x-2)^{-1/2}(6)$
$y' = \frac{3}{\sqrt{6x-2}}$

6. $y' = 2\sqrt{3x-1} + 2x(\frac{1}{2})(3x-1)^{-1/2}(3)$
$y' = 2\sqrt{3x-1} + \frac{3x}{\sqrt{3x-1}}$

7. $y = (\cos(2x))^3$
$y' = 3\cos^2(2x)(-\sin(2x))(2)$
$y' = -6\cos^2(2x)\sin(2x)$

8. $f'(x) = 3x^2\cos(3x) + x^3(-\sin(3x))(3)$
$f'(x) = 3x^2\cos(3x) - 3x^3\sin(3x)$

9. $f'(x) = \frac{4}{3}(3x^2-2x)^{1/3}(6x-2)$
$f'(x) = (8x - \frac{8}{3})(3x^2-2x)^{1/3}$

10. $f'(x) = 2\tan(5x)\sec^2(5x)(5)$
$f'(x) = 10\tan(5x)\sec^2(5x)$

11. $1 - dy/dx = y + x\,dy/dx$
$1 - y = x\,dy/dx + dy/dx$
$1 - y = dy/dx(x+1)$
$dy/dx = \frac{1-y}{x+1}$

12. $2x + 2(y + x\,dy/dx) + 3y^2\,dy/dx = 0$
$2x + 2y + 2x\,dy/dx + 3y^2\,dy/dx = 0$
$2x\,dy/dx + 3y^2\,dy/dx = -2x - 2y$
$dy/dx = \frac{-2x-2y}{2x+3y^2}$

13. $1 + \cos y \, dy/dx = 2y \, dy/dx$

$1 + \cos y = 2y \, dy/dx - dy/dx$

$dy/dx = \frac{1+\cos y}{2y-1}$

14. $\sec^2 y \, dy/dx = 1 - dy/dx$

$\sec^2 y \, dy/dx + dy/dx = 1$

$dy/dx(\sec^2 y + 1) = 1$

$dy/dx = \frac{1}{\sec^2 y + 1}$

15. $f'(x) = 35x^4 - 16x^3 + 24x^2 - 2$

16. $f'(x) = x^{-2/3} - x^{-1/2} - x^{-2}$

$f'(x) = \frac{1}{x^{2/3}} - \frac{1}{x^{1/2}} - \frac{1}{x^2}$

17. $f'(x) = \frac{3(2x+1) - 2(3x-2)}{(2x+1)^2}$ Simplify

$f'(x) = \frac{6x+3-6x+4}{(2x+1)^2} = \frac{7}{(2x+1)^2}$

18. $f'(x) = -\csc x \cot x \tan x + \csc x \sec^2 x$

$= -\csc x + \csc x \sec^2 x$

$f'(x) = \csc x(\sec^2 x - 1)$

19. $y' = \frac{-\sin(2x)(2)\sin(3x) - \cos(2x)\cos(3x)(3)}{\sin^2(3x)}$

$\frac{\cos(2x)}{\sin(3x)}$

$y' = \frac{-2\sin(2x)\sin(3x) - 3\cos(2x)\cos(3x)}{\sin^2(3x)}$

20. $y = (x-2)^2 \cos x$

$y' = 2(x-2)\cos x + (x-2)^2(-\sin x)$

$y' = 2(x-2)\cos x - (x-2)^2 \sin x$

21. $f(x) = \frac{x^2-2}{x-1}$

$f'(x) = \frac{2x(x-1) - (x^2-2)}{(x-1)^2}$

$f'(x) = \frac{2x^2 - 2x - x^2 + 2}{(x-1)^2}$

$f'(x) = \frac{x^2 - 2x + 2}{(x-1)^2}$

22. $g(s) = (s^5 - 4)(s^3 + 3) = s^8 + 3s^5 - 4s^3 - 12$ much easier if this way!
$g'(s) = 8s^7 + 15s^4 - 12s^2$

23. $f(x) = 4x + 6x^{1/2}$
$f'(x) = 4 + 3x^{-1/2}$
$f'(x) = 4 + 3/\sqrt{x}$

24. $f(x) = -5x^{-9}$
$f'(x) = 45x^{-10}$
$f'(x) = 45/x^{10}$

25. $f(x) = x^{4/3}$
$f'(x) = \frac{4}{3}x^{1/3}$

26. $f(x) = x^{1/3} + x^{5/4}$
$f'(x) = \frac{1}{3}x^{-2/3} + 5/4\,x^{1/4}$
$f'(x) = \frac{1}{3x^{2/3}} + \frac{5x^{1/4}}{4}$

27. $f(x) = 3x$ @ $x = -2$
$f'(x) = 3$ $f'(-2) = 3$

28. $f(x) = \frac{1}{4}x^{-4} + \frac{1}{2}x^{-2} + x$ @ $x = 2$
$f'(x) = -\frac{1}{x^5} - \frac{1}{x^3} + 1$
$f'(2) = \frac{1}{2^5} - \frac{1}{2^3} + 1 = \frac{1}{32} - \frac{1}{8} + 1$
$= -\frac{3}{32} + 1 = 29/32$

29. $g(x) = 3(\sin x)^2$
$g'(x) = 6\sin x \cos x$
$g'(\pi/6) = 6\sin\pi/6\cos\pi/6 = 6(1/2)(\sqrt{3}/2) = 3\sqrt{3}/2$

30. $x^2 + y^2 = 7 - xy \quad (3,-2)$

$2x + 2y\,dy/dx = -(y + x\,dy/dx)$

$2y\,dy/dx + x\,dy/dx = -2x - y$

$dy/dx = \frac{-2x-y}{2y+x}$ @ $(3,-2)$ $dy/dx = \frac{-2(3)+2}{2(-2)+3} = \frac{-4}{-1} = \boxed{4}$

31. $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

32. $f(x) = (4x^2 - 5)^3$

$f'(x) = 3(4x^2-5)^2(8x) = 24x(4x^2-5)^2$

$f''(x) = 24(4x^2-5)^2 + 24x(2)(4x^2-5)(8x)$

$f''(x) = 24(4x^2-5)^2 + 384x^2(4x^2-5)$

33. $f(x) = x^2(2x+1)^3$ @ $x = -1 \quad (-1,-1)$

$f'(x) = 2x(2x+1)^3 + x^2(3)(2x+1)^2(2)$

$f'(x) = 2x(2x+1)^3 + 6x^2(2x+1)^2$

$f'(-1) = -2(-1)^3 + 6(-1)^2 = 2 + 6 = 8$

$y + 1 = 8(x+1)$ or $y = 8x + 7$

34. $2x + 2y\,dy/dx = 0$

$dy/dx = -\frac{x}{y} = 8/15$

$y - 15 = 8/15(x + 8)$

35. $f(x) = 2x^5 + 3x^4 - 7x^3 + 5x^2 - 10x + 21$

$f'(x) = 10x^4 + 12x^3 - 21x^2 + 10x - 10$

$f''(x) = 40x^3 + 36x^2 - 42x + 10$

$f'''(x) = 120x^2 + 72x - 42$

$f^{iv}(x) = 240x + 72$

36. $f(x) = (2x+5)^{1/2}$ $\quad g(x) = f^{-1}(x)$

$f'(x) = \frac{1}{\sqrt{2x+5}}$ $\quad g: (3,2)$ $\quad f: (2,3)$

$f'(2) = 1/3 \rightarrow g'(3) = 3$

37. $f(x) = \sin x$ $\quad g: (\frac{1}{2}, \frac{\pi}{6})$ $\quad f: (\pi/6, 1/2)$

$f'(x) = \cos x$

$f'(\pi/6) = \cos(\pi/6) = \sqrt{3}/2$ $\quad g'(1/2) = 2/\sqrt{3} = 2\sqrt{3}/3$

39. $y = \operatorname{arccsc}(2x)$

$y' = \frac{-2}{|2x|\sqrt{4x^2-1}} = \frac{-1}{|x|\sqrt{4x^2-1}}$

40. $y = \operatorname{arccot}(x+1)$

$y' = \frac{1}{1+(x+1)^2} \overset{OR}{=} \frac{1}{x^2+2x+2}$

41. $y = \arcsin(2x)$

$y' = \frac{2}{\sqrt{1-4x^2}}$

42. $y = \arctan(3x)$

$y' = \frac{3}{1+9x^2}$

43. $y = \arcsin(x^{1/2})$ $\quad u = x^{1/2}$ $\quad u' = \frac{1}{2\sqrt{x}}$

$y' = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}}$

$y' = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{\sqrt{x}\sqrt{1-x}}{2x(1-x)}$

44. $y = \operatorname{arcsec}(e^{2x})$

$y' = \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}}$

45. $y = \arcsin x + x(1-x^2)^{1/2}$

$y' = \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + x\left(\frac{1}{2\sqrt{1-x^2}}\right)(-2x)$

$y' = \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$

$= \frac{1+(1-x^2)-x^2}{\sqrt{1-x^2}} = \frac{2(1-x^2)}{\sqrt{1-x^2}}$

$= \frac{-2(x^2-1)\sqrt{1-x^2}}{1-x^2}$

$= \frac{2(x^2-1)\sqrt{1-x^2}}{x^2-1}$

$\boxed{= 2\sqrt{1-x^2}}$

and That's my final answer!!

46. $y = \frac{1}{2}\arccos(x)$ $\quad \left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$

$y' = \frac{-1}{2\sqrt{1-x^2}}$ @ $-\frac{\sqrt{2}}{2} = \frac{-1}{2\sqrt{\frac{1}{2}}} = \frac{-1}{2/\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$y - 3\pi/8 = -\frac{\sqrt{2}}{2}\left(x + \sqrt{2}/2\right)$

48. $f(x) = x^3 + 2x - 1$ $\quad g:(2,1)\quad f:(1,2)$

$f'(x) = 3x^2 + 2$

$f'(1) = 5 \longrightarrow \boxed{g'(2) = 1/5}$

49. $f(x) = \frac{x+3}{x-5}$ $\quad g:(-7,4)\quad f(4,-7)$

$f'(x) = \frac{x-5-(x+3)}{(x-5)^2}$

$= \frac{-2}{(x-5)^2}$

$f'(4) = -2 \longrightarrow \boxed{g'(-7) = -1/2}$

50. $f(x) = (2x+5)^{1/2}$ $\quad g:(3,2)\quad f:(2,3)$

$f'(x) = \frac{2}{2\sqrt{2x+5}} = \frac{x}{\sqrt{2x+5}}$

$f'(2) = \frac{2}{3} \longrightarrow \boxed{g'(3) = \frac{3}{2}}$

51. $f(x) = \sin x$ $\quad g:(\frac{1}{2}, \frac{\pi}{6})\quad f:(\frac{\pi}{6}, \frac{1}{2})$

$f'(x) = \cos x$

$f'(\pi/6) = \sqrt{3}/2 \longrightarrow \boxed{g'(\frac{1}{2}) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}}$

52. $f(x) = (x-4)^{\frac{1}{2}}$ $\quad g:(1,5)\quad f:(5,1)$

$f'(x) = \frac{1}{2\sqrt{x-4}}$

$f'(5) = \frac{1}{2} \longrightarrow \boxed{g'(1) = 2}$

53. $y = \arccos(x^{1/2})$

$y' = \frac{-\frac{1}{2\sqrt{x}}}{\sqrt{1-x}} = -\frac{1}{2\sqrt{x}\sqrt{1-x}} = -\frac{\sqrt{x}\sqrt{1-x}}{2x(1-x)}$

54. $y = x(1-x^2)^{\frac{1}{2}}$

$y' = (1-x^2)^{1/2} + x\left(\frac{1}{2}\right)(-2x)(1-x^2)^{-1/2}$

$= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} = \frac{1-x^2-x^2}{\sqrt{1-x^2}} = \frac{(1-2x^2)\sqrt{1-x^2}}{1-x^2}$

55. $f(x) = \log_5 x(3x+2)^{1/2}$ $\quad u = x(3x+2)^{1/2}$

$u' = \sqrt{3x+2} + \dfrac{3x}{2\sqrt{3x+2}}$

$f'(x) = \dfrac{9x+4}{2\sqrt{3x+2}}$

$= \dfrac{2(3x+2)+3x}{2\sqrt{3x+2}} = \dfrac{9x+4}{2\sqrt{3x+2}}$

$x\sqrt{3x+2}\ln 5$

$= \dfrac{9x+4}{2\sqrt{3x+2}} \cdot \dfrac{1}{x\sqrt{3x+2}\ln 5}$

$$\boxed{f'(x) = \frac{9x+4}{2x(3x+2)\ln 5}}$$

56. $f(x) = \ln(2x^2+1)$ $\quad u = 2x^2+1$

$f'(x) = \dfrac{4x}{2x^2+1}$ $\quad u' = 4x$

57. $y = \arcsin(e^{3x})$ $\quad u = e^{3x}$

$y' = \dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$ $\quad u' = 3e^{3x}$

$u^2 = e^{6x}$

58. $y = 5^{4x^2}$ $\quad u = 4x^2$

$\boxed{y' = (5^{4x^2})(\ln 5)(8x)}$ $\quad u' = 8x$

59. $y = 3x\arcsin x$

$y' = 3\arcsin x + 3x\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$y' = 3\arcsin x + \dfrac{3x}{\sqrt{1-x^2}}$ ok to leave like this

Use log properties to re-write.

60. $y = \ln\left(\frac{5x^2}{x+4}\right)$

$y = \ln 5x^2 - \ln(x+4)$

$y = \ln 5 + 2\ln x - \ln(x+4)$

$y' = \frac{2}{x} - \frac{1}{x+4} = \frac{2(x+4) - x}{x(x+4)} = \boxed{\frac{x+8}{x(x+4)}}$

61. $y = 2\arccos x \quad \left(\frac{1}{2}, \frac{2\pi}{3}\right)$

$y' = \frac{-2}{\sqrt{1-x^2}} \quad y'(\frac{1}{2}) = \frac{-2}{\sqrt{\frac{3}{4}}} = \frac{-2}{\frac{\sqrt{3}}{2}} = -\frac{4}{\sqrt{3}} = -\frac{4\sqrt{3}}{3} = m$

$\boxed{y - \frac{2\pi}{3} = -\frac{4\sqrt{3}}{3}\left(x - \frac{1}{2}\right)}$