AP Calculus AB

Name KEY

Practice 3.3 – 3.4

Find the derivative of each function.

1) $y = (3x^4 + 5)^3$

$y' = 3(3x^4+5)^2(12x^3)$

$\boxed{y' = 36x^3(3x^4+5)^2}$

or

$y' = 324x^{11} + 1080x^7 + 900x^3$

2) $y = \sec\frac{x}{5}$

$\boxed{y' = \frac{1}{5}\sec\frac{x}{5}\tan\frac{x}{5}}$

3) $f(x) = \frac{-2}{(3-x^2)^3}$ $f(x) = -2(3-x^2)^{-3}$

$f'(x) = \frac{6(-2x)}{(3-x^2)^4}$

$\boxed{f'(x) = \frac{-12x}{(3-x^2)^4}}$

4) $f(x) = \frac{3x}{(4x^2-7)^3}$ $f(x) = 3x(4x^2-7)^{-3}$

Product Rule

$f'(x) = (4x^2-7)^{-3}(3) + (3x)(-3)(4x^2-7)^{-4}(8x)$

$f'(x) = \frac{3}{(4x^2-7)^3} + \frac{-72x}{(4x^2-7)^4}$

$f'(x) = \frac{3(4x^2-7)^1 - 72x}{(4x^2-7)^4}$

product rule $f'(x) = \frac{12x^2-72x-21}{(4x^2-7)^4} = \boxed{\frac{3(4x^2-24x-7)}{(4x^2-7)^4}}$

5) $y = \sqrt{6x-2}$ $y = (6x-2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(6x-2)^{-\frac{1}{2}}(6)$

$\boxed{y' = \frac{3}{\sqrt{6x-2}}}$

6) $y = 2x\sqrt{3x-1}$ $y = 2x(3x-1)^{\frac{1}{2}}$

$y' = (3x-1)^{\frac{1}{2}}(2) + (2x)(\frac{1}{2})(3x-1)^{-\frac{1}{2}}(3)$

$y' = \frac{2(3x-1)^{\frac{1}{2}}}{(3x-1)^{\frac{1}{2}}} + \frac{3x}{(3x-1)^{\frac{1}{2}}}$ $(3x-1)^{\frac{1}{2}}$

$y' = \frac{2(3x-1)+3x}{(3x-1)^{\frac{1}{2}}} = \frac{9x-2}{(3x-1)^{\frac{1}{2}}} = \boxed{\frac{9x-2}{\sqrt{3x-1}}}$

7) $y = \cos^3(2x)$

$y' = 3[\cos(2x)]^2(-\sin(2x))(2)$

$\boxed{y' = -6\cos^2(2x)\sin(2x)}$

product rule 8) $f(x) = x^3\cos(3x)$

$f'(x) = \cos(3x)(3x^2) + (x^3)(-\sin 3x)(3)$

$f'(x) = 3x^2\cos(3x) - 3x^3\sin 3x$

$\boxed{f'(x) = 3x^2(\cos(3x) - x\sin(3x))}$

KEY

9) $f(x) = (3x^2 - 2x)^{\frac{4}{3}}$

$f'(x) = \frac{4}{3}(3x^2-2x)^{\frac{1}{3}}(6x-2)$

$\boxed{f'(x) = \left(8x - \frac{8}{3}\right)\sqrt[3]{3x^2-2x}}$

10) $f(x) = \tan^2(5x)$

$f'(x) = 2\tan(5x)\sec^2(5x)(5)$

$\boxed{f'(x) = 10\tan(5x)\sec^2(5x)}$

quotient rule

11) $f(x) = \frac{3x-2}{2x+1}$

$f'(x) = \frac{(2x+1)(3) - (3x-2)(2)}{(2x+1)^2}$

$f'(x) = \frac{6x+3-6x+4}{(2x+1)^2}$

$\boxed{f'(x) = \frac{7}{(2x+1)^2}}$

product rule

12) $f(x) = \csc x \cdot \tan x$

$f'(x) = \tan x(-\csc x \cot x) + \csc x(\sec^2 x)$

$f'(x) = -\csc x + \csc x \sec^2 x$

$f'(x) = \csc x(\sec^2 x - 1)$

$f'(x) = \csc x(\tan^2 x)$

$f'(x) = \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos^2 x}\right)$

$f'(x) = \frac{\sin x}{\cos^2 x}$

$\boxed{f'(x) = \tan x \sec x}$

product rule

13) $y = (x-2)^2 \cos x$

$y' = \cos x(2)(x-2)^1(1) + (x-2)^2(-\sin x)$

$y' = 2(x-2)\cos x - (x-2)^2 \sin x$

$\boxed{y' = (x-2)(2\cos x - (x-2)\sin x)}$

14) $g(s) = (s^5 - 4)(s^3 + 3)$

$g'(s) = (s^3+3)(5s^4) + (s^5-4)(3s^2)$

$g'(s) = 5s^7 + 15s^4 + 3s^7 - 12s^2$

$\boxed{g'(s) = 8s^7 + 15s^4 - 12s^2}$

product rule

15) $g(x) = e^x(x^3 + 2)$

$g'(x) = (x^3+2)(e^x) + e^x(3x^2)$

$\boxed{g'(x) = e^x(x^3+2+3x^2)}$

16) $y = \sin(e^x)$

$y' = \cos e^x (e^x)$

$\boxed{y' = e^x \cos e^x}$

17) Find the first four derivatives of $f(x) = 2x^5 + 3x^4 - 7x^3 + 5x^2 - 10x + 21$.

$f'(x) = 10x^4 + 12x^3 - 21x^2 + 10x - 10$

$f''(x) = 40x^3 + 36x^2 - 42x + 10$

$f'''(x) = 120x^2 + 72x - 42$

$f^4(x) = 240x + 72$

quotient rule

#4) $\dfrac{(4x^2-7)^{\cancel{3}\,1}(3) - (3x)(3)\cancel{(4x^2-7)^2}(8x)}{(4x^2-7)^{\cancel{6}\,4}}$

$$f'(x) = \frac{(4x^2-7)(3) - 72x}{(4x^2-7)^4} = \frac{12x^2-72x-21}{(4x^2-7)^4} = \boxed{\frac{3(4x^2-24x-7)}{(4x^2-7)^4}}$$