CALCULUS BC
WORKSHEET ON PARAMETRIC EQUATIONS AND GRAPHING

Work these on notebook paper. Make a table of values and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Do not use your calculator.

1. $x = 2t + 1$ and $y = t - 1$

2. $x = 2t$ and $y = t^2$, $-1 \le t \le 2$

3. $x = 2 - t^2$ and $y = t$

4. $x = \sqrt{t + 2}$ and $y = 3 - t$

5. $x = t - 2$ and $y = 1 - \sqrt{t}$

6. $x = 2t$ and $y = |t - 1|$

7. $x = t$ and $y = \dfrac{1}{t^2}$

8. $x = 2\cos t - 1$ and $y = 3\sin t + 1$

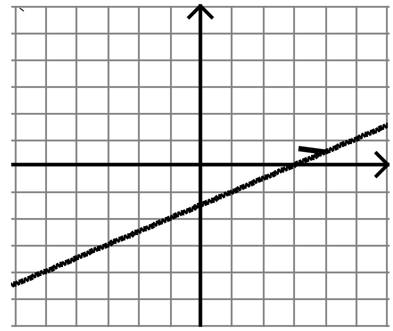
9. $x = 2\sin t - 1$ and $y = \cos t + 2$

10. $x = \sec t$ and $y = \tan t$

Answers to Worksheet on Parametric Equations and Graphing

1. $x = 2t + 1$ and $y = t - 1$

t	-2	-1	0	1	2
x	-3	-1	1	3	5
y	-3	-2	-1	0	1



To eliminate the parameter, solve for $t = \frac{1}{2}x - \frac{1}{2}$.

Substitute into y's equation to get $y = \frac{1}{2}x - \frac{3}{2}$.

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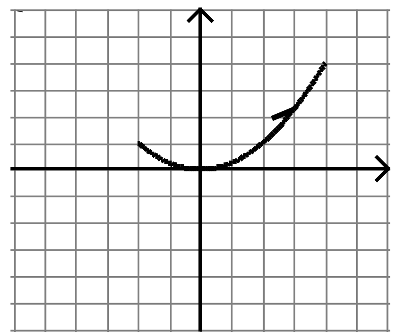
2. $x = 2t$ and $y = t^2$, $-1 \le t \le 2$

t	-1	0	1	2
x	-2	0	2	4
y	1	0	1	4



To eliminate the parameter, solve for $t = \frac{x}{2}$.

Substitute into y's equation to get

$y = \frac{x^2}{4}$, $-2 \le x \le 4$. Note: The restriction on x

is needed for the graph of $y = \frac{x^2}{4}$ to match the parametric graph.

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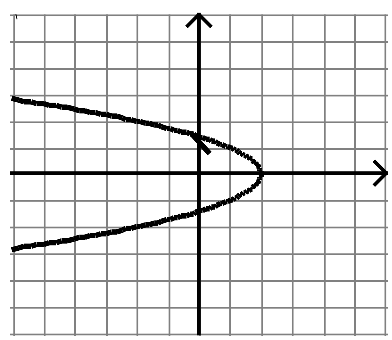
3. $x = 2 - t^2$ and $y = t$

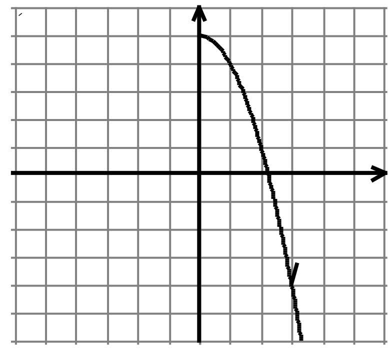
t	-2	-1	0	1	2
x	-2	1	2	1	-2
y	-2	-1	0	1	2



To eliminate the parameter, notice that $t = y$.
Substitute into x's equation to get
$x = 2 - y^2$.

4. $x=\sqrt{t+2}$ and $y=3-t$

t	-2	-1	2	7
x	0	1	2	3
y	5	4	1	-4



To eliminate the parameter, solve for $t=x^2-2$.
Substitute into y's equation to get
$y=5-x^2,\ x\geq 0$. Note: The restriction on x is needed for the graph of $y=5-x^2$ to match the parametric graph.

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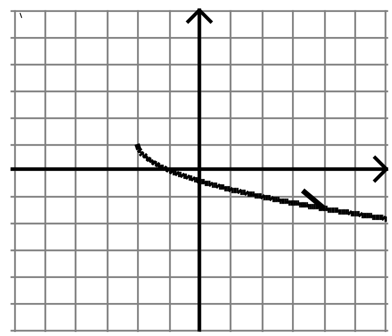
5. $x=t-2$ and $y=1-\sqrt{t}$

t	0	1	4	9
x	-2	-1	2	7
y	1	0	-1	-2



To eliminate the parameter, solve for $t=x+2,\ x\geq -2$ (since $t \geq 0$). Substitute into y's equation to get $y=1-\sqrt{x+2}$.

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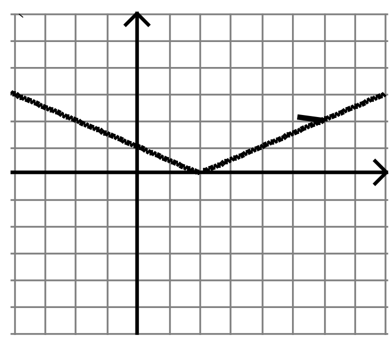
6. $x=2t$ and $y=|t-1|$

t	-2	-1	0	1	2	3
x	-1	-2	0	2	4	6
y	3	2	1	0	1	2



To eliminate the parameter, solve for $t=\dfrac{x}{2}$.

Substitute into y's equation to get

$y=\left|\dfrac{x}{2}-1\right|$ or $y=\dfrac{|x-2|}{2}$.

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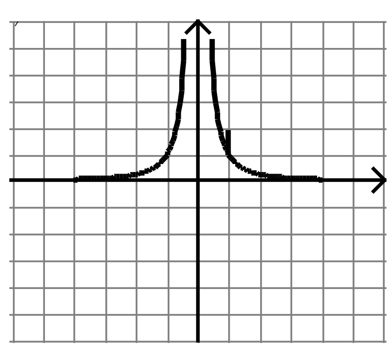
7. $x=t$ and $y=\dfrac{1}{t^2}$

t	-2	-1	$-1/2$	0	1/2	1	2
x	-2	-1	$-1/2$	0	1/2	1	2
y	1/4	1	4	und.	4	1	1/4

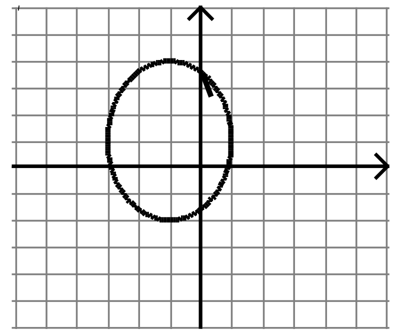


To eliminate the parameter, notice that $t=x$.

Substitute into y's equation to get $y=\dfrac{1}{x^2}$.

8. $x = 2\cos t - 1$ and $y = 3\sin t + 1$

t	0	π/2	π	3π/2	2π
x	1	−1	−3	−1	1
y	1	4	1	−2	1



To eliminate the parameter, solve for $\cos t$ in x's equation and $\sin t$ in y's equation. Substitute into the trigonometric identity

$\cos^2 t + \sin^2 t = 1$ to get $\dfrac{(x+1)^2}{4} + \dfrac{(y-1)^2}{9} = 1$.

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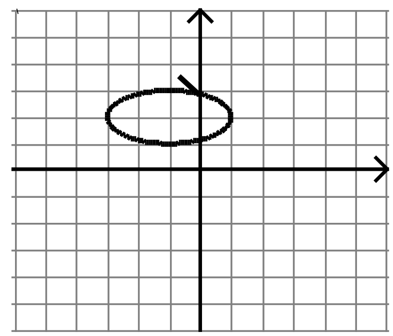
9. $x = 2\sin t - 1$ and $y = \cos t + 2$

t	0	π/2	π	3π/2	2π
x	−1	1	−1	−3	−1
y	3	2	1	2	3



To eliminate the parameter, solve for ☐ in y's equation and ☐ in x's equation. Substitute into the trigonometric identity

☐ to get ☐.

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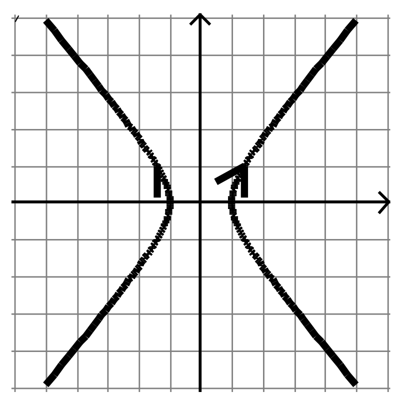
10. $x = \sec t$ and $y = \tan t$

t	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
x	1	$\sqrt{2}$	und.	$-\sqrt{2}$	−1	$-\sqrt{2}$	und.	$\sqrt{2}$	1
y	0	1	und.	−1	0	1	und.	-1	0



To eliminate the parameter, substitute into the trigonometric identity $1 + \tan^2 t = \sec^2 t$ to get $1 + y^2 = x^2$ or $x^2 - y^2 = 1$.

CALCULUS BC
WORKSHEET ON PARAMETRICS AND CALCULUS

Work these on **notebook paper**. Do not use your calculator.

On problems 1 – 5, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $x = t^2,\ y = t^2 + 6t + 5$

2. $x = t^2 + 1,\ y = 2t^3 - t^2$

3. $x = \sqrt{t},\ y = 3t^2 + 2t$

4. $x = \ln t,\ y = t^2 + t$

5. $x = 3\sin t + 2,\ y = 4\cos t - 1$

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6. A curve C is defined by the parametric equations $x = t^2 + t - 1,\ y = t^3 - t^2$.
   (a) Find $\frac{dy}{dx}$ in terms of t.
   (b) Find an equation of the tangent line to C at the point where $t = 2$.

7. A curve C is defined by the parametric equations $x = 2\cos t,\ y = 3\sin t$.
   (a) Find $\frac{dy}{dx}$ in terms of t.
   (b) Find an equation of the tangent line to C at the point where $t = \frac{\pi}{4}$.

On problems 8 – 10, find:
(a) $\frac{dy}{dx}$ in terms of t.
(b) all points of horizontal and vertical tangency

8. $x = t + 5,\ y = t^2 - 4t$

9. $x = t^2 - t + 1,\ y = t^3 - 3t$

10. $x = 3 + 2\cos t,\ y = -1 + 4\sin t$

On problems 11 - 12, a curve C is defined by the parametric equations given. For each problem, write an integral expression that represents the length of the arc of the curve over the given interval.

11. $x = t^2,\ y = t^3,\ 0 \le t \le 2$

12. $x = e^{2t} + 1,\ y = 3t - 1,\ -2 \le t \le 2$

Answers to Worksheet on Parametrics and Calculus

1. $\frac{dy}{dx} = \frac{2t+6}{2t} = 1 + \frac{3}{t}$; $\frac{d^2y}{dx^2} = \frac{-\frac{3}{t^2}}{2t} = -\frac{3}{2t^3}$

2. $\frac{dy}{dt} = 3t - 1$; $\frac{d^2y}{dx^2} = \frac{3}{2t}$

3. $\frac{dy}{dx} = \frac{6t+2}{\frac{1}{2}t^{-\frac{1}{2}}} = 12t^{\frac{3}{2}} + 4t^{\frac{1}{2}}$; $\frac{d^2y}{dx^2} = \frac{18t^{\frac{1}{2}} + 2t^{-\frac{1}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}} = 36t + 4$

4. $\frac{dy}{dx} = \frac{2t+1}{\frac{1}{t}} = 2t^2 + t$; $\frac{d^2y}{dx^2} = \frac{4t+1}{\frac{1}{t}} = 4t^2 + t$

5. $\frac{dy}{dx} = \frac{-4\sin t}{3\cos t} = -\frac{4}{3}\tan t$; $\frac{d^2y}{dx^2} = \frac{-\frac{4}{3}\sec^2 t}{3\cos t} = -\frac{4}{9}\sec^3 t$

6. (a) $\frac{dy}{dx} = \frac{3t^2 - 2t}{2t+1}$ (b) $y - 4 = \frac{8}{5}(x-5)$

7. (a) $\frac{dy}{dx} = \frac{3\cos t}{-2\sin t} = -\frac{3}{2}\cot t$ (b) $y - \frac{3\sqrt{2}}{2} = -\frac{3}{2}\left(x - \sqrt{2}\right)$

8. (a) $\frac{dy}{dx} = \frac{2t-4}{1}$ (b) Vert. tangent at $(7, -4)$. No point of horiz. tangency on this curve.

9. (a)

(b) Vert. tangent at the points $(1, -2)$ and $(3, 2)$. Horiz. tangent at $\left(\frac{3}{4}, -\frac{11}{8}\right)$.

10. (a) $\frac{dy}{dx} = \frac{4\cos t}{-2\sin t} = -2\cot t$

(b) Vert. tangent at $(3, 3)$ and $(3, -5)$. Horiz. tangent at $(5, -1)$ and $(1, -1)$.

11. $s = \int_0^2 \sqrt{4t^2 + 9t^4}\, dt$

12. $s = \int_{-2}^2 \sqrt{4e^{4t} + 9}\, dt$

CALCULUS BC
WORKSHEET 1 ON VECTORS

Work the following on **notebook paper**. Use your calculator on problems 10 and 13c only.

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.
2. If a particle moves in the *xy*-plane so that at any time $t > 0$, its position vector is $\left\langle \ln\left(t^2 + 5t\right),\ 3t^2 \right\rangle$, find its velocity vector at time $t = 2$.
3. A particle moves in the *xy*-plane so that at any time t, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.
4. If a particle moves in the *xy*-plane so that at time t its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.
5. A particle moves on the curve ☐ so that its *x*-component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point (1, 0). Find the position of the particle at time $t = 1$.
6. A particle moves in the *xy*-plane in such a way that its velocity vector is $\left\langle 1 + t,\ t^3 \right\rangle$. If the position vector at $t = 0$ is ☐, find the position of the particle at $t = 2$.
7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?
8. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?
9. A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of C at the point $(8, -4)$.
10. A particle moves in the *xy*-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$ Find the velocity vector at the time when the particle's horizontal position is $x = 25$.
11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

    (a) Find the magnitude of the velocity vector at time $t = 5$.

    (b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$. (c) Find $\frac{dy}{dx}$ as a function of x.
12. Point $P(x,\ y)$ moves in the *xy*-plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.

    (a) Find the coordinates of P in terms of t given that $t = 1$, $x = \ln 2$, and $y = 0$.
    (b) Write an equation expressing y in terms of x.
    (c) Find the average rate of change of y with respect to x as t varies from 0 to 4.
    (d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.

(a) Find $\frac{dy}{dx}$ as a function of t. (b) Find the equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) The curve C intersects the y-axis twice. Approximate the length of the curve between the two y-intercepts.

<u>Answers to Worksheet 1 on Vectors</u>

1. $\frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t} = \frac{3te^{t^3}}{2}$	2. $\left\langle \frac{9}{14}, 12 \right\rangle$	3. $\langle 20, 24 \rangle$
4. $\langle -3, 3\pi \rangle$	5. $\left(\frac{5}{2}, \ln\left(\frac{5}{2}\right) \right)$	6. $(9, 4)$
7. $-\frac{6}{5}$	8. $t = 3$	9. $y + 4 = -\frac{1}{12}(x - 8)$
10. $\langle 7.008, -2.228 \rangle$		
11. (a) $\sqrt{2600}$ or $10\sqrt{26}$	(b) $\frac{2}{3}\left(26^{3/2} - 1\right)$	(c) $t = \sqrt{x+3}$
12. (a) $\left(\ln(t+1), t^2 - 1\right)$	(b) $y = \left(e^x - 1\right)^2 - 1$ or $y = e^{2x} - 2e^x$.	
(c) $\frac{16}{\ln 5}$	(d) 4	
13. (a) $\frac{2}{3}\cot t$	(b) $y - \left(3 + \sqrt{2}\right) = \frac{2}{3}\left(x - \left(2 - \frac{3\sqrt{2}}{2}\right)\right)$	(c) 3.756

CALCULUS BC
WORKSHEET 2 ON VECTORS

Work the following on **notebook paper**. Use your calculator on problems 7 – 12 only.

1. If $x = e^{2t}$ and $y = \sin(3t)$, find $\dfrac{dy}{dx}$ in terms of t.
2. Write an integral expression to represent the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^2 t$ for $0 \le t \le \dfrac{\pi}{2}$.
3. For what value(s) of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?
4. For any time $\square$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, find the acceleration vector.
5. Find the equation of the tangent line to the curve given by the parametric equations $x(t) = 3t^2 - 4t + 2$ and $y(t) = t^3 - 4t$ at the point on the curve where $t = 1$.
6. If $x(t) = e^t + 1$ and $y = 2e^{2t}$ are the equations of the path of a particle moving in the xy-plane, write an equation for the path of the particle in terms of x and y.
7. A particle moves in the xy-plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?
8. The position of a particle at time $\square$ is given by the parametric equations $x(t) = \dfrac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.
   (a) Find the magnitude of the velocity vector at $t = 1$.
   (b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.
   (c) When is the particle at rest? What is its position at that time?
9. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time with $\dfrac{dx}{dt} = 1 + \tan(t^2)$ and $\dfrac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.
10. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = t + \cos t$ and $y(t) = 3t + 2\sin t$, $0 \le t \le \pi$. Find the velocity vector when the particle's vertical position is $y = 5$.
11. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\dfrac{dx}{dt} = 2\sin(t^3)$ and $\dfrac{dy}{dt} = \cos(t^2)$ for $0 \le t \le 4$. At time $t = 1$, the object is at the position (3, 4).
    (a) Write an equation for the line tangent to the curve at (3, 4).
    (b) Find the speed of the object at time $t = 2$.
    (c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
    (d) Find the position of the object at time $t = 2$.

12. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = \arcsin\left(\frac{t}{t+4}\right)$ and $\frac{dy}{dt} = \ln\left(t^2+3\right)$. At time $t = 1$, the particle is at the position (5, 6).

(a) Find the speed of the object at time $t = 2$.

(b) Find the total distance traveled by the object over the time interval $1 \le t \le 2$.

(c) Find $y(2)$.

(d) For $0 \le t \le 3$, there is a point on the curve where the line tangent to the curve has slope 8. At what time t, $0 \le t \le 3$, is the particle at this point? Find the acceleration vector at this point.

<u>Answers to Worksheet 2 on Vectors</u>

1. $\frac{3\cos(3t)}{2e^{2t}}$

2.

3.

4. $v(t) = \left\langle 2t,\ \frac{2}{2t+3} \right\rangle,\ a(t) = \left(2,\ -\frac{4}{(2t+3)^2} \right)$

5. $y+3 = -\frac{1}{2}(x-1)$

6. $y = 2(x-1)^2,\ x > 1,$ or $y = 2x^2 - 4x + 2,\ x > 1$

7. 12.304

8. (a) (b) 3.816 (c) At rest when $t = 2$. Position = (4, 0)

9. $a(5) = \langle 10.178,\ 6.277 \rangle$, speed = 28.083

10. $t = 1.079,\ \langle 0.119,\ 3.944 \rangle$

11. (a) $y - 4 = 0.321(x-3)$ (b) 2.084 (c) 1.126 (d) (3.436, 3.557)

12. (a) 1.975 (b) 1.683 (c) 7.661 (d) $\langle 0.422,\ 0.179 \rangle$

CALCULUS BC
WORKSHEET 3 ON VECTORS

Work the following on **notebook paper**. Use your calculator only on problems 3 – 7.

1. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2,\ y(t) = \frac{2}{3}t^3.$

(a) Find the magnitude of the velocity vector at $t = 2$.
(b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.
(c) Find $\frac{dy}{dx}$ as a function of x.
(d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

2. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with the velocity vector $v(t) = \left(\frac{1}{t+1},\ 2t \right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector.
(b) Write an equation for the line tangent to the curve when $t = 1$.
(c) Find the magnitude of the velocity vector when $t = 1$.
(d) At what time $t > 0$ does the line tangent to the particle at $(x(t),\ y(t))$ have a slope of 12?

3. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \leq t \leq 10$.

(a) Is the particle moving to the left or to the right when $t = 2.4$? Explain your answer.
(b) Find the velocity vector at the time when the particle's vertical position is $y = 7$.

4. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(-5,\ 4)$.

(a) Find the x-coordinate of the position at time $t = 3$.
(b) For any $t \geq 0$, the line tangent to the curve at $(x(t),\ y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

5. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = e^{\cos t}$ and $\frac{dy}{dt} = \sin(t^2)$ for $0 \leq t \leq 3$. At time $t = 3$, the object is at the point (1, 4).

(a) Find the equation of the tangent line to the curve at the point where $t = 3$.
(b) Find the speed of the object at $t = 3$.
(c) Find the total distance traveled by the object over the time interval $2 \leq t \leq 3$.
(d) Find the position of the object at time $t = 2$.

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6. A particle moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with

$\dfrac{dx}{dt}=\sqrt{t^3+4}$ and $\dfrac{dy}{dt}=\cos^{-1}\left(e^{-t}\right)$. At time $t = 2$, the particle is at the point (5, 3).

(a) Find the acceleration vector for the particle at $t = 2$.
(b) Find the equation of the tangent line to the curve at the point where $t = 2$.
(c) Find the magnitude of the velocity vector at $t = 2$.
(d) Find the position of the particle at time $t = 1$.

7. An object moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with

$\dfrac{dy}{dt}=2+\sin\left(e^{t}\right)$. The derivative $\dfrac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point (4, 5).

(a) Find the y-coordinate of the position at time $t = 1$.
(b) At time $t = 3$, the value of $\dfrac{dy}{dx}$ is -1.8. Find the value of $\dfrac{dx}{dt}$ when $t = 3$.
(c) Find the speed of the object at time $t = 3$.

<u>Answers to Worksheet 3 on Vectors</u>

1. (a) (b)

   (c) $\dfrac{dy}{dx}=t=\sqrt{x+2}$ (d) $\left\langle 2,\ 4\sqrt{2}\right\rangle$

2. (a) $\left(\ln\left|t+1\right|,\ t^2+3\right)$ (b)

   (c) $\dfrac{\sqrt{17}}{2}$ (d) $t=2$

3. $\langle -0.968,\ 5.704\rangle$

4. (a) -3.996 (b) $\langle -1.746,\ -6.741\rangle$

5. (a) $y-2=1.109(x-3)$ (b) 0.555

   (c) 0.878 (d) (0.529, 4.031)

6. (a) $\langle 1.732,\ 0.137\rangle$ (b) $y-3=0.414(x-5)$

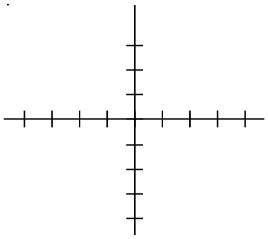
   (c) 3.750 (d) (2.239, 1.664)

7. (a) 1.269 (b) (c) 3.368

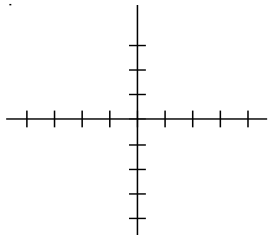
POLAR GRAPHS

Put your graphing calculator in **POLAR** mode and **RADIAN** mode. Graph the following equations on your calculator, sketch the graphs on this sheet, and answer the questions.

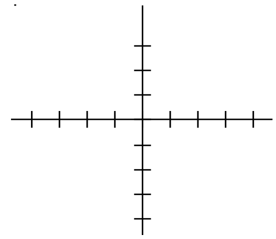
1. $r = 2\cos\theta$



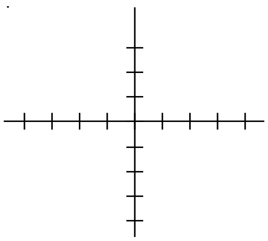
$r = 3\cos\theta$



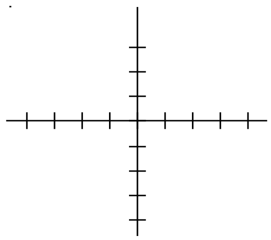
$r = -3\cos\theta$



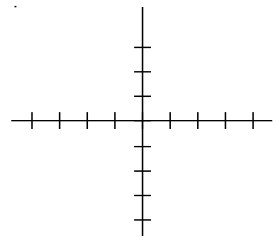
$r = 2\sin\theta$



$r = 3\sin\theta$



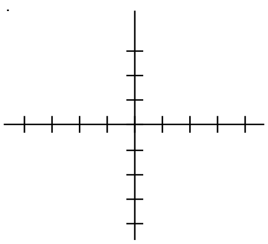
$r = -3\sin\theta$



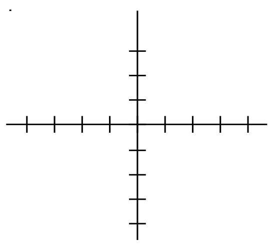
What do you notice about these graphs?

---

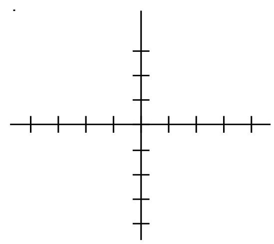
2. $r = 2 + 2\cos\theta$



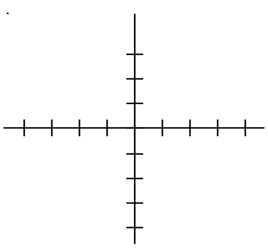
$r = 1 + 2\cos\theta$



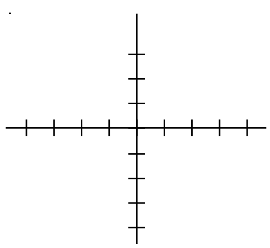
$r = 2 + \cos\theta$



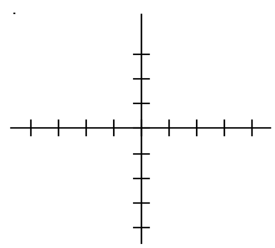
$r = 2 + 2\sin\theta$



$r = 1 + 2\sin\theta$



$r = 2 + \sin\theta$

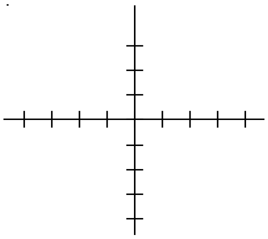
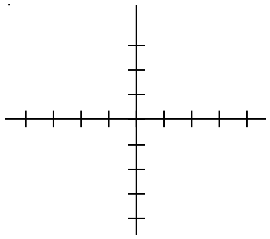
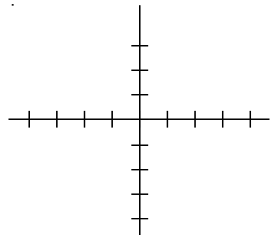
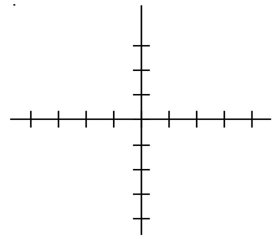


Which graphs go through the origin?

Which ones do not go through the origin?

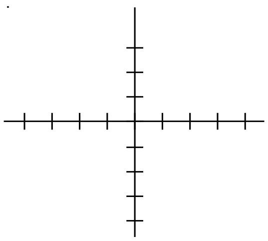
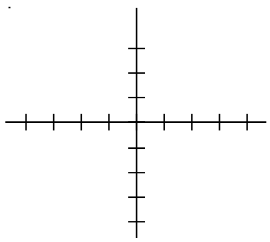
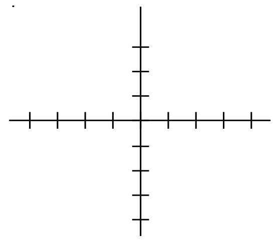
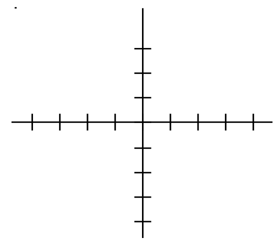
Which ones have an inner loop?

3. $r = 2\cos(3\theta)$ $\quad r = 3\cos(5\theta)$ $\quad r = 2\sin(3\theta)$ $\quad r = 3\sin(5\theta)$

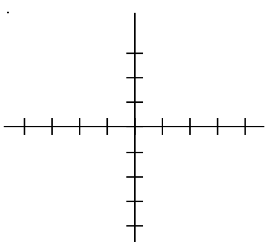
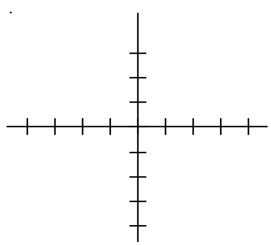
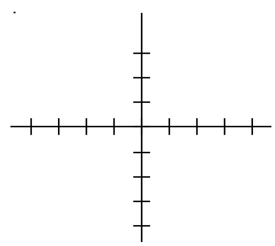
What do you notice about these graphs?

4. $r = 3\cos(2\theta)$ $\quad r = 2\cos(4\theta)$ $\quad r = 3\sin(2\theta)$ $\quad r = 2\sin(4\theta)$

What do you notice about these graphs?

5. $r^2 = 4\cos(2\theta)$ $\quad r^2 = 4\sin(2\theta)$ $\quad r = \theta$

What do you notice about these graphs?

CALCULUS BC
WORKSHEET 1 ON POLAR

Work the following on **notebook paper**. Do **NOT** use your calculator.
Convert the following equations to polar form.

1. $y = 4$ 2. $3x - 5y + 2 = 0$ 3. $x^2 + y^2 = 25$

Convert the following equations to rectangular form.

4. $r = 3\sec\theta$ 5. $r = 2\sin\theta$ 6. $\theta = \frac{5\pi}{6}$

For the following, find $\frac{dy}{dx}$ for the given value of θ.

7. $r = 2 + 3\sin\theta,\ \theta = \frac{3\pi}{2}$

8. $r = 3(1 - \cos\theta),\ \theta = \frac{\pi}{2}$

9. $r = 4\sin\theta,\ \theta = \frac{\pi}{3}$

10. $r = 2\sin(3\theta),\ \theta = \frac{\pi}{4}$

---

11. Find the points of horizontal and vertical tangency for $r = 1 + \sin\theta$. Give your answers in polar form, $(r,\ \theta)$.

Make a table, tell what type of graph (circle, cardioid, limacon, lemniscate, rose), and sketch the graph.

12. $r = 3\cos\theta$

13. $r = -2\sin\theta$

14. $r = 2 + 2\sin\theta$

15. $r = 3 + 2\cos\theta$

16. $r^2 = 4\sin(2\theta)$

17. $r = 1 + 2\sin\theta$

18. $r = 4\cos(2\theta)$

19. $r = 6\sin(3\theta)$

Answers

1. $r=\dfrac{4}{\sin\theta}$ or $r=4\csc\theta$

2. $r=\dfrac{-2}{3\cos\theta-5\sin\theta}$

3. $r = 5$

4. $x = 3$

5. $x^2+y^2=2y$

6. $y=-\dfrac{\sqrt{3}}{3}x$

7. 0

8. – 1

9. $-\sqrt{3}$

10. $\dfrac{1}{2}$

11. Horiz: $\left(2, \dfrac{\pi}{2}\right), \left(\dfrac{1}{2}, \dfrac{7\pi}{6}\right), \left(\dfrac{1}{2}, \dfrac{11\pi}{6}\right)$

    Vert.: $\left(\dfrac{3}{2}, \dfrac{\pi}{6}\right), \left(\dfrac{3}{2}, \dfrac{5\pi}{6}\right)$

12. circle centered on the x-axis with diameter 3

13. circle centered on the y-axis with diameter 2

14. cardioid with y-axis symmetry

15. limacon without a loop with x-axis symmetry

16. lemniscate

17. limacon with a loop with y-axis symmetry

18. rose with 4 petals

19. rose with 3 petals

CALCULUS BC
WORKSHEET 2 ON POLAR

Work the following on **notebook paper**.
On problems 1 – 5, sketch a graph, shade the region, set up the integrals needed, and then find the area.
Do **not** use your calculator.

1. Area of one petal of $r = 2\cos(3\theta)$

2. Area of one petal of $r = 4\sin(2\theta)$

3. Area of the interior of $r = 2 + 2\cos\theta$

4. Area of the interior of $r = 2 - \sin\theta$

5. Area of the interior of $r^2 = 4\sin(2\theta)$

On problems 6 – 7, sketch a graph, shade the region, set up the integrals needed, and then use your **calculator** to evaluate.

6. Area of the inner loop of $r = 1 + 2\cos\theta$

7. Area between the loops of $r = 1 + 2\cos\theta$

Answers to Worksheet 2 on Polar

1. Area = $\frac{1}{2}\int_{-\pi/6}^{\pi/6}\left(2\cos(3\theta)\right)^2 d\theta = \int_0^{\pi/6} 4\cos^2(3\theta)\,d\theta = \ ... \ = \frac{\pi}{3}$

2. Area = $\frac{1}{2}\int_0^{\pi/2}\left(4\sin(2\theta)\right)^2 d\theta = 8\int_0^{\pi/2}\sin^2(2\theta)\,d\theta = \ ... \ = 2\pi$

3. Area = $\frac{1}{2}\int_0^{2\pi}\left(2+2\cos\theta\right)^2 d\theta = \ ... \ = 6\pi$

4. Area = $\frac{1}{2}\int_0^{2\pi}\left(2-\sin\theta\right)^2 d\theta = ... \ = \frac{9\pi}{2}$

5. Area = $\int_0^{\pi/2} 4\sin(2\theta)\,d\theta = \ ... \ = 4$

6. Area = $\frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left(1+2\cos\theta\right)^2 d\theta = \int_{2\pi/3}^{\pi}\left(1+2\cos\theta\right)^2 d\theta = \pi - \frac{3\sqrt{3}}{2}$ or 0.544

7. Top half: Area = $\frac{1}{2}\int_0^{2\pi/3}\left(1+2\cos\theta\right)^2 d\theta - \frac{1}{2}\int_{2\pi/3}^{\pi}\left(1+2\cos\theta\right)^2 d\theta$

Between the loops:

$$\text{Area} = 2(\text{Top half}) = 2\left(\frac{1}{2}\int_0^{2\pi/3}\left(1+2\cos\theta\right)^2 d\theta - \frac{1}{2}\int_{2\pi/3}^{\pi}\left(1+2\cos\theta\right)^2 d\theta\right) = \pi + 3\sqrt{3} \text{ or } 8.338$$

OR Area = $\frac{1}{2}\int_0^{2\pi}\left(1+2\cos\theta\right)^2 d\theta - 2(\text{Answer to 6}) = \pi + 3\sqrt{3}$ or 8.338

CALCULUS BC
WORKSHEET 3 ON POLAR

Work the following on **notebook paper**.
On problems 1 – 2, sketch a graph, shade the region, set up the integrals needed, and then find the area.
Do **not** use your calculator.

1. Area inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

2. Area of the common interior of $r = 4\sin\theta$ and $r = 2$

On problems 3 – 5, sketch a graph, shade the region, set up the integrals needed, and then use your **calculator** to evaluate.

3. Area inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$

4. Area of the common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$

5. Area of the common interior of $r = 4\sin(2\theta)$ and $r = 2$

Do not use your calculator on problem 6.

6. Given $x = \sqrt{t}$ and $y = 3t^2 + 2t$, find ☐.

Use your calculator on problem 7.

7. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin(e^t)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At time $t = 3$, the object is at position $(5,\ 4)$.

(a) Find the y-coordinate of the position at time $t = 1$.

(b) For $t = 3$, the line tangent to the curve at $(x(t),\ y(t))$ has a slope of -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

(c) Find the speed of the particle when $t = 3$.

Answers to Worksheet 3 on Polar

1. Area $= \int_0^{\pi/3} (3\cos\theta)^2\, d\theta - \int_0^{\pi/3} (2-\cos\theta)^2\, d\theta = \ldots = 3\sqrt{3}$

2. Area $= \int_0^{\pi/6} (4\sin\theta)^2\, d\theta + \int_{\pi/6}^{\pi/2} (2)^2\, d\theta = \ldots = \frac{8\pi}{3} - 2\sqrt{3}$

3. Area $= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2\, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1+\sin\theta)^2\, d\theta = \pi$

4. Area $= \int_0^{\pi/3} (1+\cos\theta)^2\, d\theta + \int_{\pi/3}^{\pi/2} (3\cos\theta)^2\, d\theta = \frac{5\pi}{4}$ or 3.927

5. Area in Quad. $1 = \frac{1}{2}\int_0^{\pi/12} (4\sin(2\theta))^2\, d\theta + \frac{1}{2}\int_{\pi/12}^{5\pi/12} (2)^2\, d\theta + \frac{1}{2}\int_{5\pi/12}^{\pi/2} (4\sin(2\theta))^2\, d\theta$

Total area = $\frac{16\pi}{3} - 4\sqrt{3}$ or 9.827

6. $\frac{dy}{dx} = 12t^{3/2} + 4t^{1/2}$

$\frac{d^2y}{dx^2} = 36t + 4$

7. (a) 0.269

(b) -1.636

(c) 3.368

CALCULUS BC
WORKSHEET 4 ON POLAR

Work the following on **notebook paper**. Do **not** use your calculator on problems 1, 2, and 5.

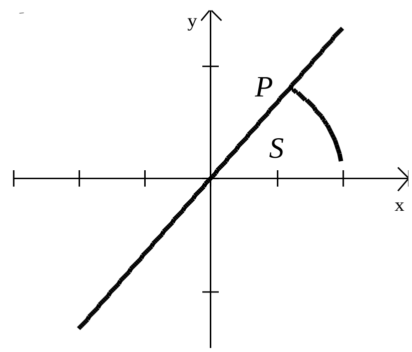
1. Sketch a graph, shade the region, and find the area inside $r = 2$ and outside $r = 2 - \sin\theta$.

---

2. Given $r = 4\sin\theta$, find $\dfrac{dy}{dx}$ when $\theta = \dfrac{\pi}{3}$.

You may use your calculator on problems 3 and 4.

3. The figure shows the graphs of the line $y = \dfrac{2}{3}x$ and the curve C given by $y = \sqrt{1 - \dfrac{x^2}{4}}$. Let S be the region in the first quadrant bounded by the two graphs and the x-axis. The line and the curve intersect at point P.



(a) Find a polar equation to represent curve C.
(b) Find the polar coordinates of point P.
(c) Find the value of $\dfrac{dr}{d\theta}$ at point P. What does your answer tell you about r? What does it tell you about the curve?
(a) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle θ that gives the area of S.

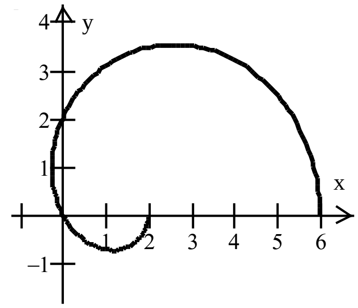
---

4. A curve is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \cos(3\theta)$ for $\dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.
(a) Find the area bounded by the curve and the y-axis.
(b) Find the angle θ that corresponds to the point on the curve with y-coordinate -1.
(c) For what values of θ, $\pi \le \theta \le \dfrac{3\pi}{2}$, is $\dfrac{dr}{d\theta}$ positive? What does this say about r? What does it say about the curve?
(d) Find the value of θ on the interval $\pi \le \theta \le \dfrac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is the greatest distance? Justify your answer.
(e) A particle is traveling along the polar curve given by $r = \theta + \cos(3\theta)$ so that its position at time t is $(x(t),\ y(t))$ and such that $\dfrac{d\theta}{dt} = 2$. Find the value of $\dfrac{dy}{dt}$ at the instant that $\theta = \dfrac{7\pi}{6}$, and interpret the meaning of your answer in the context of the problem.

TURN->>>

Do **not** use your calculator on problem 5.

5. The graph of the polar curve $r = 2 + 4\cos\theta$ for $0 \le \theta \le \pi$ is shown on the right. Let S be the shaded region in the fourth quadrant bounded by the curve and the x-axis.



(a) Write an expression for $\dfrac{dy}{d\theta}$ in terms of θ.

(b) A particle is traveling along the polar curve given by $r = 2 + 4\cos\theta$ so that its position at time t is $\left(x(t),\ y(t)\right)$ and such that $\dfrac{d\theta}{dt} = -2$. Find the value of $\dfrac{dy}{dt}$ at the instant that $\theta = \dfrac{\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

Use your calculator on problem 6.

6. An object moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with $\dfrac{dx}{dt} = 2\sin\left(t^3\right)$ and $\dfrac{dy}{dt} = \cos\left(t^2\right)$ for $0 \le t \le 3$. At time $t = 1$, the object is at the point (3, 4).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.

(b) Find the speed of the object at $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 2$.

Answers to Worksheet 4 on Polar

1. Area $= \dfrac{1}{2}\displaystyle\int_0^{\pi}\left(2^2 - (2-\sin\theta)^2\right)d\theta = \ldots = 4 - \dfrac{\pi}{4}$

2. $\dfrac{dy}{dx} = -\sqrt{3}$

3. (a) $r = \sqrt{\dfrac{4}{4\sin^2\theta + \cos^2\theta}}$ (b) (1.442, 0.588)

(c) $\dfrac{dr}{d\theta} = -1.038$ so r is decreasing, and the curve is moving closer to the origin. (d) 0.927

4. (a) 19.675 (b) 3.485

(c) $\dfrac{dr}{d\theta} > 0$ for $(\pi,\ 4.302)$. This means that the r is getting larger, and the curve is getting farther from the origin.

(d)

θ	r
π	2.142
4.302	5.245
$\dfrac{3\pi}{2}$	4.712

The greatest distance is 5.245 when θ = 4.302.

(e) $\dfrac{dy}{dt} = -10.348$. This means that the y-coordinate is decreasing at a rate of 10.348.

5. (a) $\dfrac{dy}{d\theta} = 2\cos\theta + 4\cos^2\theta - 4\sin^2\theta$ (b) $\dfrac{dy}{dt} = 2$. When $\theta = \dfrac{\pi}{3}$, the y-coordinate is increasing at a rate of 2.

6. (a) $y - 4 = 0.321(x - 3)$ (b) 2.084 (c) 1.126 (d) $\langle 3.436, 3.557\rangle$

AP CALCULUS BC
REVIEW SHEET FOR TEST ON PARAMETRICS, VECTORS, POLAR, & AP REVIEW

Use your calculator on problems 2 – 3 and 9. Show supporting work, and give decimal answers correct to three decimal places.

1. Find given $x = t^2 + 1,\ y = 2t^3 - t^2$.

---

2. An object moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with

$\frac{dx}{dt} = \sin\left(t^3\right)$ and $\frac{dy}{dt} = \cos\left(t^2\right)$. At time $t = 2$, the object is at the position (7, 4).

(a) Write an equation for the line tangent to the curve at the point where $t = 2$.
(b) Find the speed of the object at time $t = 2$.
(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
(d) For what value of $t,\ 0 < t < 1,$ does the tangent line to the curve have a slope of 4? Find the acceleration vector at this time.
(e) Find the position of the object at time $t = 1$.

---

3. An object moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with

$\frac{dx}{dt} = 1 + \sin\left(t^3\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At $t = 2$, the object is at the point $(-5, 4)$.

(a) Find the x-coordinate of the position at time $t = 3$.
(b) For any $t \ge 0$, the line tangent to the curve at $\left(x(t),\ y(t)\right)$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

No calculator.

4. Find $\frac{dy}{dx}$ for the given value of θ given $r = 4\sin\theta,\ \theta = \frac{\pi}{3}$.

No calculator.

5. Find the area of the interior of $r = 2 + 2\cos\theta$.
6. Find the area of one petal of $r = 2\cos(3\theta)$.
7. Set up the integral(s) needed to find the area inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$. Do not evaluate.
8. Set up the integral(s) needed to find the area of the common interior of $r = 4\sin\theta$ and $r = 2$. Do not evaluate.

Use your calculator.

9. A curve is drawn in the xy-plane and is described by the equation in polar coordinates

$r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.

(a) Find the area bounded by the curve and the y-axis.
(b) Find the angle θ that corresponds to the point on the curve with y-coordinate -1.

Answers

1. $\dfrac{dy}{dx}=3t-1; \quad \dfrac{d^2y}{dx^2}=\dfrac{3}{2t}$

2. (a) $y-4=-0.661(x-7)$ (b) 1.186 (c) 0.976

(d) $t = 0.6164\ldots,\ a(0.616)=\langle 1.109, -0.457\rangle$ (e) $\langle 6.782, 4.443\rangle$

3. (a) – 3.996 (b) $\langle -1.746, -6.741\rangle$

4. $-\sqrt{3}$

5. $A=\dfrac{1}{2}\int_0^{2\pi}(2+2\cos\theta)^2\,d\theta = \ldots = 6\pi$

6. $A=\dfrac{1}{2}\int_{-\pi/6}^{\pi/6}(2\cos(3\theta))^2\,d\theta = \ldots = \dfrac{\pi}{3}$

7. Top half doubled: $A=\int_0^{\pi/3}(3\cos\theta)^2 d\theta-\int_0^{\pi/3}(2-\cos\theta)^2 d\theta$

8. Right side doubled: $A=\int_0^{\pi/6}(4\sin\theta)^2 d\theta+\int_{\pi/6}^{\pi/2}(2)^2 d\theta$

9. (a) $A=\dfrac{1}{2}\int_{\pi/2}^{3\pi/2}(\theta+\cos(3\theta))^2 d\theta = 19.675\,19.675$

(b) $(\theta+\cos(3\theta))(\sin\theta)=-1$

$\theta=3.485$