

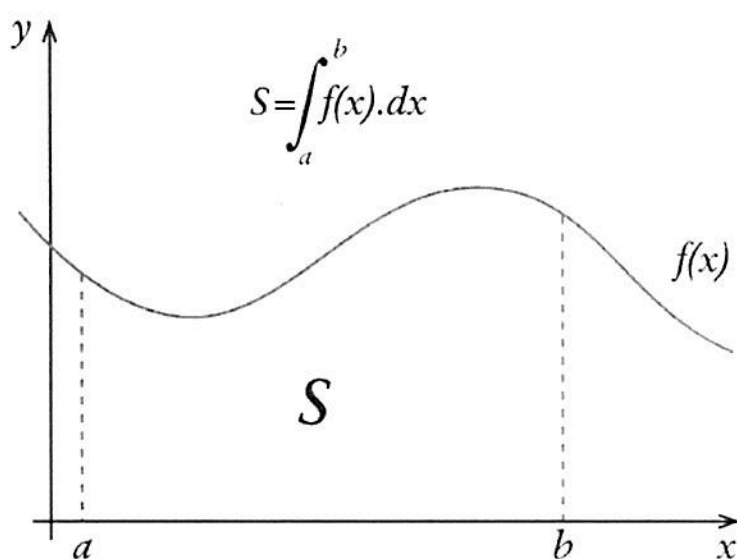
The Official Guide to Calculus

2015-2016

It's

Beyond **C**omprehension

(But **A**lmost **B**elievable)



AP CALCULUS
Stuff you **MUST** Know Cold

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined.

local minimum :

$\frac{dy}{dx}$ goes $(-, 0, +)$ or $(-, \text{und}, +)$

or $\frac{d^2y}{dx^2} > 0$.

local maximum :

$\frac{dy}{dx}$ goes $(+, 0, -)$ or $(+, \text{und}, -)$

or $\frac{d^2y}{dx^2} < 0$.

pt of inflection : concavity changes.

$\frac{d^2y}{dx^2}$ goes $(+, 0, -)$, $(-, 0, +)$,

$(+, \text{und}, -)$, or $(-, \text{und}, +)$

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

More Derivatives

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Differentiation Rules

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$.

Corollary to FTC

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(t)dt =$$

$$f(b(x))b'(x) - f(a(x))a'(x)$$

Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, then for any number c between $f(a)$ and $f(b)$, there exists a number d in the open interval (a, b) such that $f(d) = c$.

Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, the first derivative exist on the interval (a, b) , and $f(a) = f(b)$; then there exists a number $x = c$ on (a, b) such that

$$f'(c) = 0.$$

Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and the first derivative exists on the interval (a, b) , then there exists a number $x = c$ on (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem of the Mean Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exist on the interval (a, b) , then there exists a number $x = c$ on (a, b) such that

$$f'(c) = \frac{\int_a^b f(x)dx}{(b-a)}.$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{b-a}{2n} [f(x_0)$$

$$+ 2f(x_1) + \dots$$

$$+ 2f(x_{n-1}) + f(x_n)]$$

Solids of Revolution and friends

Disk Method

$$V = \pi \int_a^b [R(x)]^2 dx$$

Washer Method

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$$

Shell Method (no longer on AP)

$$V = 2\pi \int_a^b r(x)h(x)dx$$

ArcLength

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Surface of revolution (No longer on AP)

$$S = 2\pi \int_a^b r(x) \sqrt{1 + [f'(x)]^2} dx$$

Distance, velocity and acceleration

velocity = $\frac{d}{dt}$ (position).

acceleration = $\frac{d}{dt}$ (velocity).

velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$.

speed = $|v| = \sqrt{(x')^2 + (y')^2}$.

$$\text{Distance} = \int_{\text{initial time}}^{\text{final time}} |v| dt$$

$$= \int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2} dt$$

average velocity = $\frac{\text{final position} - \text{initial position}}{\text{total time}}.$

Integration by Parts

$$\int u dv = uv - \int v du$$

Integral of Log

$$\int \ln x dx = x \ln x - x + C.$$

Taylor Series

If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial

$$\begin{aligned} f(x) &\approx f(a) + f'(a)(x-a) \\ &\quad + \frac{f''(a)}{2!}(x-a)^2 + \dots \\ &\quad + \frac{f^{(n)}(a)}{n!}(x-a)^n. \end{aligned}$$

Maclaurin Series

A Taylor Series about $x = 0$ is called Maclaurin.

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots \\ \cos(x) &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \dots \\ \sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \\ \frac{1}{1-x} &= 1 + x + x^2 + x^3 + \dots \\ \ln(x+1) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \end{aligned}$$

Lagrange Error Bound

If $P_n(x)$ is the n^{th} degree Taylor polynomial of $f(x)$ about c and $|f^{(n+1)}(t)| \leq M$ for all t between x and c , then

$$|f(x) - P_n(x)| \leq \frac{M}{(n+1)!} |x - c|^{n+1}$$

Alternating Series Error Bound

If $S_N = \sum_{k=1}^N (-1)^k a_k$ is the N^{th} partial sum of a convergent alternating series, then

$$|S_\infty - S_N| \leq |a_{N+1}|$$

Euler's Method

If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through (x_0, y_0) ,
 $y(x_0) = y_0$

$$\vdots$$

$$y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$$

In other words:

$$x_{\text{new}} = x_{\text{old}} + \Delta x$$

$$y_{\text{new}} = y_{\text{old}} + \left. \frac{dy}{dx} \right|_{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$$

Ratio Test

The series $\sum_{k=0}^{\infty} a_k$ converges if

$$\lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| < 1.$$

If limit equals 1, you know nothing.

Polar Curves

For a polar curve $r(\theta)$, the Area inside a "leaf" is

$$\int_{\theta_1}^{\theta_2} \frac{1}{2} [r(\theta)]^2 d\theta,$$

where θ_1 and θ_2 are the "first" two times that $r = 0$.

The slope of $r(\theta)$ at a given θ is

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} \\ &= \frac{\frac{d}{d\theta}[r(\theta) \sin \theta]}{\frac{d}{d\theta}[r(\theta) \cos \theta]} \end{aligned}$$

l'Hopital's Rule

If $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$,
 then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$

BC Calculus – Formula Sheet

Derivatives

- 1) $\frac{d}{dx}(k) = 0$
- 2) $\frac{d}{dx}(kf(x)) = kf'(x)$
- 3) $\frac{d}{dx}(u^n) = nu^{n-1} \cdot u'$
- 4) $\frac{d}{dx}(uv) = uv' + vu'$
- 5) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
- 6) $\frac{d}{dx}[g(f(x))] = g'(f(x)) \cdot f'(x)$
- 7) $\frac{dy}{dt} = \frac{dy/dx}{dt/dx}$ or $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$
- 8) $\frac{d}{dx}(\sin u) = \cos u \cdot u'$
- 9) $\frac{d}{dx}(\cos u) = -\sin u \cdot u'$
- 10) $\frac{d}{dx}(\tan u) = \sec^2 u \cdot u'$
- 11) $\frac{d}{dx}(\cot u) = -\csc^2 u \cdot u'$
- 12) $\frac{d}{dx}(\sec u) = \sec u \tan u \cdot u'$
- 13) $\frac{d}{dx}(\csc u) = -\csc u \cot u \cdot u'$
- 14) $\frac{d}{dx}(\arcsin u) = \frac{d}{dx} \sin^{-1} u = \frac{u'}{\sqrt{1-u^2}}$
- 15) $\frac{d}{dx}(\arccos u) = \frac{d}{dx} \cos^{-1} u = \frac{-u'}{\sqrt{1-u^2}}$
- 16) $\frac{d}{dx}(\arctan u) = \frac{d}{dx} \tan^{-1} u = \frac{u'}{1+u^2}$
- 17) $\frac{d}{dx}(\text{arc cot } u) = \frac{d}{dx} \cot^{-1} u = \frac{-u'}{1+u^2}$
- 18) $\frac{d}{dx}(\text{arc sec } u) = \frac{d}{dx} \sec^{-1} u = \frac{u'}{|u|\sqrt{u^2-1}}$
- 19) $\frac{d}{dx}(\text{arc csc } u) = \frac{d}{dx} \csc^{-1} u = \frac{-u'}{|u|\sqrt{u^2-1}}$
- 20) $\frac{d}{dx}(\ln u) = \frac{u'}{u}$
- 21) $\frac{d}{dx}(e^u) = e^u \cdot u'$
- 22) $\frac{d}{dx}(a^u) = (\ln a)a^u \cdot u'$
- 23) $\frac{d}{dx}(\log_a u) = \frac{1}{\ln a} \cdot \frac{u'}{u}$

Integrals

- 1) $\int kf(x) dx = k \int f(x) dx$
- 2) $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$
- 3) $\int du = u + C$
- 4) $\int u^n du = \frac{u^{n+1}}{n+1}$
- 5) $\int \frac{du}{u} = \ln|u| + C$
- 6) $\int e^u du = e^u + C$
- 7) $\int a^u du = \frac{1}{\ln a} \cdot a^u + C$
- 8) $\int \cos u du = \sin u + C$
- 9) $\int \sin u du = -\cos u + C$
- 10) $\int \sec^2 u du = \tan u + C$
- 11) $\int \csc^2 u du = -\cot u + C$
- 12) $\int \sec u \tan u du = \sec u + C$
- 13) $\int \csc u \cot u du = -\csc u + C$
- 14) $\int \tan u du = \ln|\sec u| + C = -\ln|\cos u| + C$
- 15) $\int \cot u du = \ln|\sin u| + C = -\ln|\csc u| + C$
- 16) $\int \sec u du = \ln|\sec u + \tan u| + C$
- 17) $\int \csc u du = -\ln|\csc u + \cot u| + C$
- 18) $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
- 19) $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
- 20) $\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + C$
- 21) $\int_a^b f(x) dx = -\int_b^a f(x) dx$
- 22) $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$
- 23) Integration by parts: $\int u dv = uv - \int v du$

Miscellaneous Formulas

- 1) Average value of a function

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x) dx$$

- 2) Fundamental theorem of calculus

$$\int_a^b f'(x) dx = f(b) - f(a)$$

(Definite integral of a rate of change of a function represents net change in function on interval)

- 3) Net change in position on $a \leq t \leq b$

$$\int_a^b v(t) dt$$

- 4) Total distance traveled on $a \leq t \leq b$

$$\int_a^b |v(t)| dt$$

- 5) Trapezoidal rule:

$$T(n) = \frac{\Delta x}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

with n subintervals of length $\Delta x = \frac{b-a}{n}$

- 6) Second fundamental theorem

$$\text{If } F(x) = \int_a^x g(t) dt, \text{ then } F'(x) = g(x)$$

$$\text{and if } F(x) = \int_a^u g(t) dt, \text{ then } F'(x) = g(u) \cdot \frac{du}{dx}$$

- 7) Area between two curves

$$\int_a^b [f(x) - g(x)] dx \text{ if } f(x) \geq g(x) \text{ on } a \leq x \leq b$$

- 8) Volumes of solids of revolution (can by dx or dy , but I am writing dx for each)

$$\text{by disk: } V = \int_a^b \pi r^2 dx$$

$$\text{by washer: } V = \int_a^b \pi (R^2 - r^2) dx$$

$$\text{by shells: } V = \int_a^b 2\pi rh dx$$

- 9) Volumes of solids with known cross sections

$$V = \int_a^b A(x) dx$$

- 10) Definition of derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- 11) If f and g are inverse functions

$$f'(x) = \frac{1}{g'(f(x))}$$

- 12) Law of exponential change

$$\text{If } \frac{dy}{dt} = ky, \text{ then } y = Ce^{kt}$$

- 13) Point slope form of equation of a line

$$y - y_1 = m(x - x_1)$$

- 14) Even function - symmetric to y -axis

$$f(-x) = f(x)$$

- 15) Odd function - origin symmetry

$$f(-x) = -f(x)$$

- 16) Definitions of absolute value

$$|x| = \sqrt{x^2} = \begin{cases} x, & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

- 17) $\ln e^x = x$, $e^{\ln x} = x$

$$18) \log_a u = \frac{\ln u}{\ln a}$$

- 19) Mean Value theorem (for derivatives)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or instantaneous rate of change equals average rate of change on an interval

- 20) Vertical asymptote of $x = a$ if a will make the denominator 0 but not the numerator

- 21) Horizontal asymptote of $y = c$ if $\lim_{x \rightarrow \pm\infty} f(x) = c$

- 22) Limits of rational functions

degree of $p >$ degree of q :

$$\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = \pm\infty$$

degree of $p =$ degree of q :

$$\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = \text{ratio of leading coefficients}$$

degree of $p <$ degree of q :

$$\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = 0$$

- 23) Special Trigonometric limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

- 24) L'Hopital's Rule

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ yields an indeterminate form such as

$0/0$ or ∞/∞ , then use L'Hopital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

25) Area bounded by polar curve on $\alpha \leq \theta \leq \beta$

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$$

26) Arc length in rectangular form

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

27) Arc length in parametric form

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

28) Parametric derivatives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$$

29) Three different growth models

I. Uninhibited growth or decay

$$\frac{dQ}{dt} = kQ \rightarrow Q = Q_0 e^{kt}$$

II. Limited growth

$$\frac{dQ}{dt} = k(B - Q) \rightarrow Q = B - Ae^{kt}$$

III. Inhibited or Logistics Growth

$$\frac{dQ}{dt} = kQ(B - Q) \rightarrow Q = \frac{B}{1 + Ae^{-Bkt}}$$

30) Work: Hooke's Law

$F = kx$, x is change in spring length from natural length

$$W = \int_a^b kx dx$$

31) Work: pumping liquid

$$\Delta W = (\text{weight of slice})(\text{distance pumped})$$

$$W = \int_a^b dW \quad (dx \text{ or } dy \text{ is in weight of slice})$$

32) Two dimensional motion

Given $x(t)$ and $y(t)$,

$$\vec{v} = \frac{dx}{dt} \vec{i} + \frac{dy}{dt} \vec{j}$$

$$\vec{a} = \frac{d^2x}{dt^2} \vec{i} + \frac{d^2y}{dt^2} \vec{j}$$

$$\text{speed} = |\vec{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

33) Euler's method

$$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$$

Trigonometry

1) $\sin(-x) = -\sin x$

2) $\cos(-x) = \cos x$

3) $\sin^2 x + \cos^2 x = 1$

4) $\tan^2 x + 1 = \sec^2 x$

5) $1 + \cot^2 x = \csc^2 x$

6) $\sin 2x = 2 \sin x \cos x$

7) $\cos 2x = \cos^2 x - \sin^2 x$

$$= 2 \cos^2 x - 1$$

$$= 2 \sin^2 x - 1$$

8) $\sin^2 x = \frac{1 - \cos 2x}{2}$

9) $\cos^2 x = \frac{1 + \cos 2x}{2}$

10) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

11) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

12) Ranges for inverse trigonometric functions

$y = \sin^{-1} x \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \csc^{-1} x \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$

$y = \tan^{-1} x \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \cos^{-1} x \quad 0 \leq y \leq \pi$

$y = \sec^{-1} x \quad 0 \leq y \leq \pi, y \neq \frac{\pi}{2}$

$y = \cot^{-1} x \quad 0 < y < \pi$

Geometry

1) sphere: $V = \frac{4}{3} \pi r^3$

$$SA = 4\pi r^2$$

2) cylinder: $V = \pi r^2 h$

$$SA = 2\pi r h + 2\pi r^2$$

3) cone: $V = \frac{1}{3} \pi r^2 h$

4) equilateral Δ : $A = \frac{s^2 \sqrt{3}}{4}$

Series

1) Geometric Series Test: $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $|r| < 1$, diverges if $|r| \geq 1$

2) p -series Test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, diverges if $p \leq 1$

3) n^{th} Term Test: $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$ (cannot be used to show convergence)

4) Alternating Series Test: $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges if $0 \leq a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$

Alternating Series Remainder: maximum error $= |R_n| \leq a_{n+1}$

5) Integral Test: If f is a positive, continuous decreasing function and $f(n) = a_n$, then $\sum_{n=1}^{\infty} a_n$ and $\int f(x) dx$ either both converge or both diverge.

Integral Remainder: maximum error $= R_n \leq \int_n^{\infty} f(x) dx$

6) Root Test: $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$, diverges if $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} > 1$, inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 1$

7) Ratio Test: $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$, diverges if $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$, inconclusive if $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

8) Direct Comparison Test: If $0 \leq a_n \leq b_n$ for all n and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $0 \leq b_n \leq a_n$ for all n and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

9) Limit comparison test: If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ is finite and positive, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

Taylor Series

Taylor series for $f(x)$ (centered at $x = c$):

$$\frac{f(c)}{0!} + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Maclaurin series (Taylor series centered at $x = 0$):

$$\frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Three important Maclaurin series to know (which happen to converge for all x):

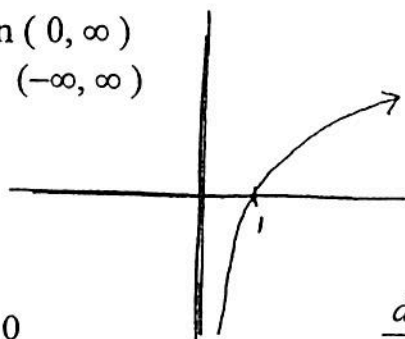
$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots + \frac{1}{n!}x^n + \dots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \dots \quad \cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots + \frac{(-1)^n}{(2n)!}x^{2n} + \dots$$

CALCULUS
CHAPTER 5 HELP SHEET
MRS. DUBNER

NATURAL LOG DEFINITION: $\ln x = \int_1^x \frac{1}{t} dt, x > 0$

$y = \ln x$ domain $(0, \infty)$
range $(-\infty, \infty)$



$$\frac{d}{dx} \ln x = \frac{1}{x}, x > 0$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \sin u du = -\cos u + c$$

$$\int \tan u du = -\ln|\cos u| + c$$

$$\int \sec u du = \ln|\sec u + \tan u| + c$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\int e^x dx = e^x + c$$

$$\int e^u du = e^u + c$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\int \frac{1}{u} du = \ln|u| + c$$

$$\int \cos u du = \sin u + c$$

$$\int \cot u du = \ln|\sin u| + c$$

$$\int \csc u du = -\ln|\csc u + \cot u| + c$$

$$\frac{d}{dx} a^x = a^x (\ln a)$$

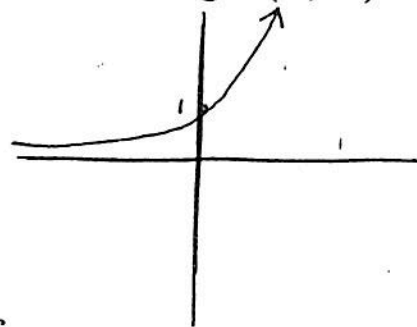
$$\frac{d}{dx} a^u = a^u (\ln a) \frac{du}{dx}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x (\ln a)}$$

$$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$y = e^x$ domain $(-\infty, \infty)$
range $(0, \infty)$



Definition $a^x = e^{(\ln a)x}$

interest compounded n times per year - $A = P \left(1 + \frac{r}{n} \right)^{nt}$

interest compounded continuously - $A = P e^{rt}$

$y = C e^{kt}$ y, differentiable function of t, $y > 0$, $y' = ky$ for constant k.

C = initial value of y

BC Calculus
Chapter 5 Formulas

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}, u > 0$$

$$\int \frac{du}{u} = \ln|u| + C \quad \text{or} \quad \int \frac{1}{u} du = \ln|u| + C$$

$$\int \sin u \, du = -\cos u + C \quad \int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \quad \int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \quad \int \csc u \, du = -\ln|\cos u + \cot u| + C$$

$$\int e^u \, du = e^u + C$$

$$\frac{d}{dx} e^x = e^x$$

$$\begin{aligned} \frac{d}{dx} a^x &= (\ln a) a^x & \frac{d}{dx} a^u &= (\ln a) a^u \cdot \frac{du}{dx} \\ \frac{d}{dx} \log_a x &= \frac{1}{(\ln a)x} & \frac{d}{dx} \log_a u &= \frac{1}{(\ln a)u} \cdot \frac{du}{dx} \end{aligned}$$

$$\int a^x \, dx = \frac{1}{\ln a} \cdot a^x + C \quad \int a^u \, du = \frac{1}{\ln a} \cdot a^u + C$$

$$\frac{d}{dx} [\arcsin u] = \frac{du}{\sqrt{1-u^2}}, |u| < 1 \quad \frac{d}{dx} [\arccos u] = \frac{-du}{\sqrt{1-u^2}}, |u| < 1$$

$$\frac{d}{dx} [\arctan u] = \frac{du}{1+u^2} \quad \frac{d}{dx} [\text{arccot } u] = \frac{-du}{1+u^2}$$

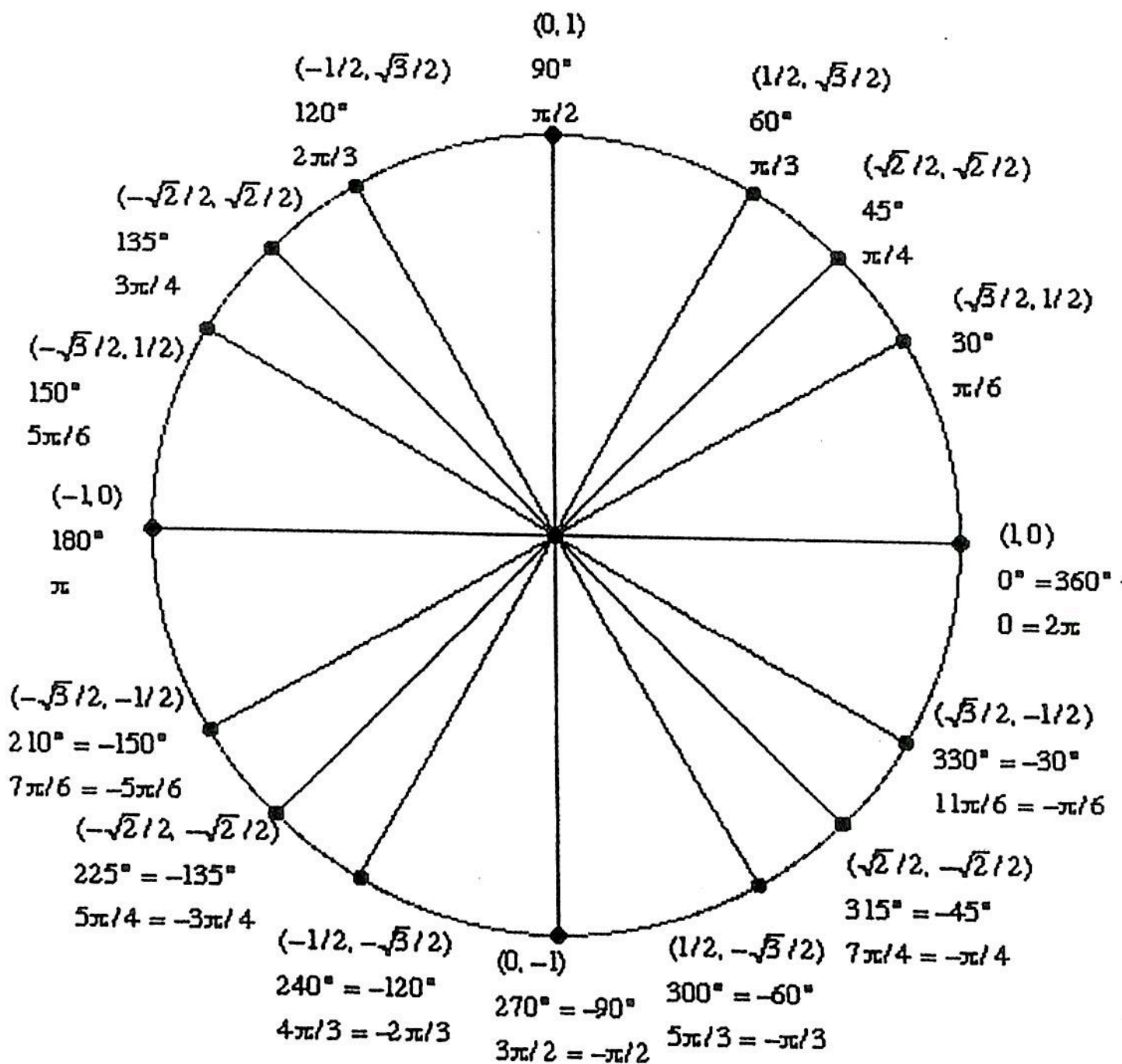
$$\frac{d}{dx} [\text{arcsec } u] = \frac{du}{|u|\sqrt{u^2-1}}, |u| > 1 \quad \frac{d}{dx} [\text{arccsc } u] = \frac{-du}{|u|\sqrt{u^2-1}}, |u| > 1$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \text{arcsec} \frac{|u|}{a} + C$$

Unit Circle Diagram



$$\sin \theta = y$$

$$\csc \theta = \frac{1}{y}$$

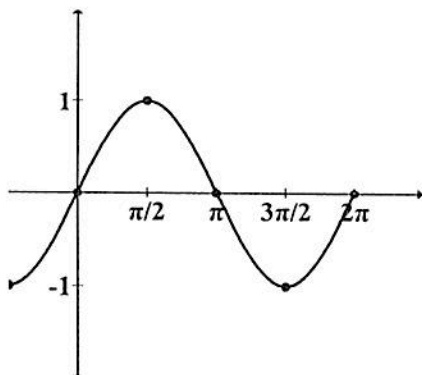
$$\cos \theta = x$$

$$\sec \theta = \frac{1}{x}$$

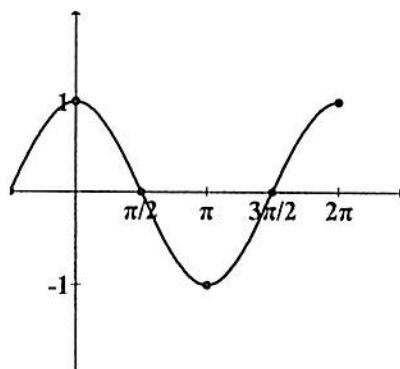
$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

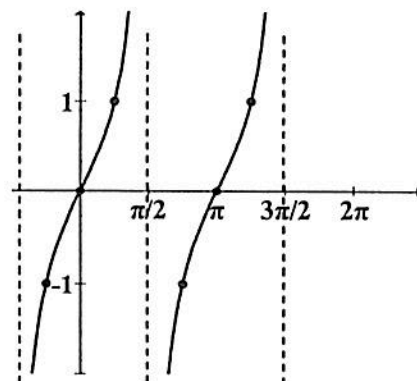
Trig Graph Help



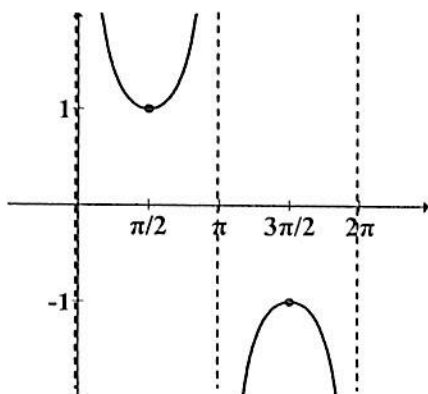
$$y = \sin(x)$$



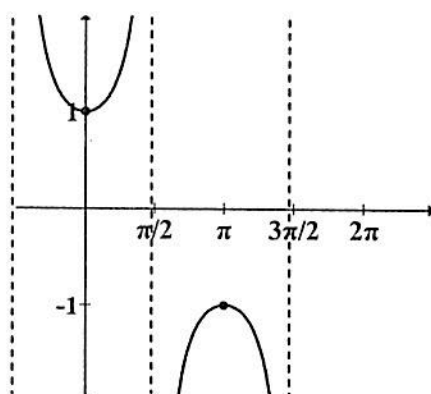
$$y = \cos(x)$$



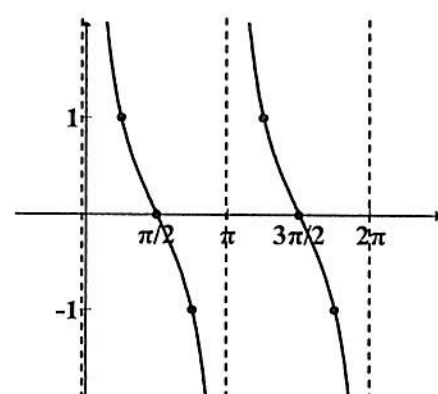
$$y = \tan(x)$$



$$y = \csc(x)$$



$$y = \sec(x)$$



$$y = \cot(x)$$

$$y = A \sin B(x - c) + D$$

$$|A| = \text{amplitude}$$

$$\frac{2\pi}{B} = \text{period}$$

$$y = A \cos B(x - c) + D$$

$$|A| = \text{amplitude}$$

$$\frac{2\pi}{B} = \text{period}$$

Period -

- Only $\sin x$ and $\cos x$ have an amplitude
- Period for $\sin x$, $\cos x$, $\sec x$, $\csc x$ is $\frac{2\pi}{B}$
- Period for $\tan x$ and $\cot x$ is $\frac{\pi}{B}$

Transformations – (Shifts)

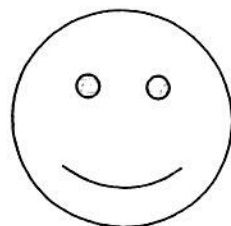
- C is the horizontal Shift
- D is the vertical shift

Asymptotes –

- $\tan x$ and $\sec x$ have asymptotes at $(2n + 1)\frac{\pi}{2}$
- $\cot x$ and $\csc x$ have asymptotes at $n\pi$

Trigonometric Identities -

Help Sheet



Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \quad \csc(-u) = -\csc u$$

$$\cos(-u) = \cos u \quad \sec(-u) = \sec u$$

$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$

Sum or Difference of Two Angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Area of a Triangle

$$\begin{aligned}\text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} ac \sin B\end{aligned}$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \text{ etc.}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Given:	Use:	To Find:
SAS	Law of Cosines	3 rd side and one remaining angle
SSS	Law of Cosines	Any two angles
ASA or AAS	Law of Sines	remaining sides
SSA	Law of Sines	an angle opp a given side and then the 3 rd side. Note: 0, 1 or 2 triangles possible

PreCalculus TRICKS AND HELPERS

Trig Post-It Matchup

This activity requires a little prep work but is a fun way to start class and get them thinking about math from the second they arrive. On small post-it notes, write an angle measure in degrees. On another, write an angle measure that is coterminal. Write enough pairs so that everyone in the class will receive an angle. Stand at the door as students enter and hand them a post-it (mix up the order first!). Once all are distributed, tell students to find their coterminal partner! I had students take their homework with them and begin to compare homework answers once they found their match. Don't forget to use both positive and negative angles! This also works great for degrees and radians.

Exact Values Shortcut #1

Write the numbers below, spaced apart, and show students how to find exact values for sin, cos, and tan by placing each number under a square root and dividing by 2.

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0
	<hr/>				
	2				

For example, the cos 45° is "2", so take the square root and divide by 2.
Sin 90° is "4", so $\sqrt{4} / 2 = 2/2 = 1$

Exact Values Shortcut #2

For the quadrantal angles, teach your students the following chant:

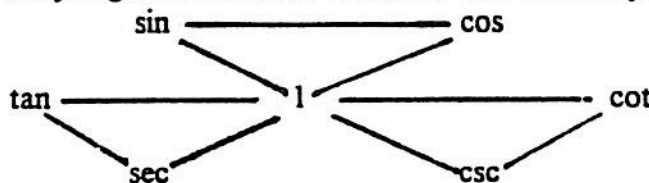
oh I owe
I owe you
oh I don't owe
I don't owe you

Pretty easy, but what does this have to do with trig? "Oh" is the number 0, "I" is the number 1, "you" means "U" as in undefined, and "don't" means negative. Here's the trig equivalent:

	sin	cos	tan
0°	0	1	0
90°	1	0	U
180°	0	-1	0
270°	-1	0	U

Identity Web

To assist with the Pythagorean identities as well as with the Reciprocals, use the following "web":



Each triangle relates the components of the Pythagorean identity (ex: $\tan^2 x + 1 = \sec^2 x$) and reciprocals are straight across from one another.

Logarithm Help Sheet

Definition: The *logarithm to base b* of a positive number x , denoted by $\log_b x$, is defined to be the exponent a that you get when you write x as a power of b . (Note: $b > 0$, $b \neq 1$)

$$\log_b x = a \text{ if and only if } x = b^a$$

The base b logarithmic function is the inverse of the base b exponential function.

Properties: M, N , positive real numbers and b positive and unequal to one.

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M = \log_b N \text{ iff } M = N$$

$$\log_b M^k = k \log_b M \text{ for any real number, } k$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\text{Simple Growth: } A_t = A_0(1 + r)^t$$

$$\text{Compound Growth: } A_t = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$$\text{Continuous Growth: } A_t = A_0 e^{rt}$$

$$\text{Half-Life: } A_t = A_0 \left(\frac{1}{2}\right)^{\frac{t}{k}}$$

Laws of Exponents

SAME BASES

$$(b^x)(b^y) = b^{x+y}$$

$$\frac{b^x}{b^y} = b^{x-y} \quad (b \neq 0)$$

If $b \neq 0, 1$, or -1 , then $b^x = b^y$ if and only if $x = y$

SAME EXPONENTS

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (b \neq 0)$$

POWER OF A POWER

$$(b^x)^y = b^{xy}$$

DEFINITIONS

$$b^0 = 1 \quad (b \neq 0)$$

$$b^{-x} = \frac{1}{b^x} \quad (x > 0 \text{ and } b \neq 0)$$

Special Polar Graphs

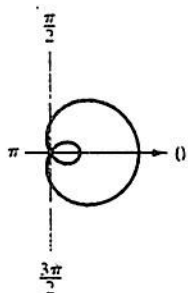
Several important types of graphs have equations that are simpler in polar form than in rectangular form. For example, the polar equation of a circle having a radius of a and centered at the origin is simply $r = a$. Later in the text you will come to appreciate this benefit. For now, we summarize some other types of graphs that have simpler equations in polar form. (Conics are considered in Section 10.5).

Limaçons

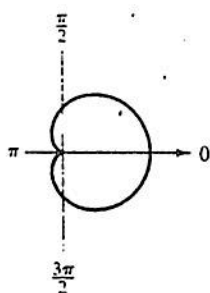
$$r = a \pm b \cos \theta$$

$$r = a \pm b \sin \theta$$

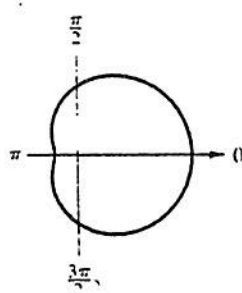
$$(0 < a, 0 < b)$$



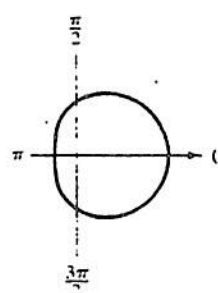
$\frac{a}{b} < 1$
Limaçon with
inner loop



$\frac{a}{b} = 1$
Cardioid
(heart-shaped)



$1 < \frac{a}{b} < 2$
Dimpled
limaçon



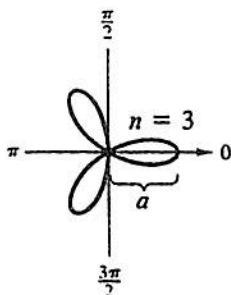
$\frac{a}{b} \geq 2$
Convex
limaçon

Rose Curves

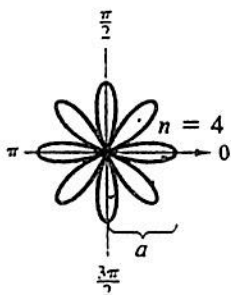
n petals if n is odd

$2n$ petals if n is even

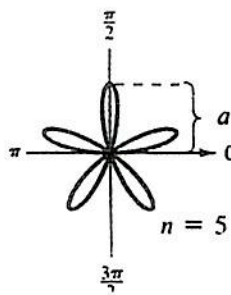
($n \geq 2$)



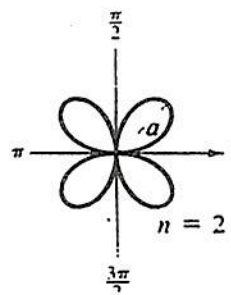
$r = a \cos n\theta$
Rose curve



$r = a \cos n\theta$
Rose curve

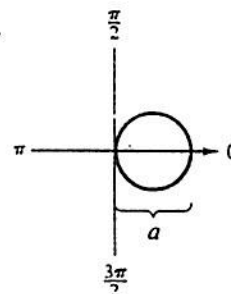


$r = a \sin n\theta$
Rose curve

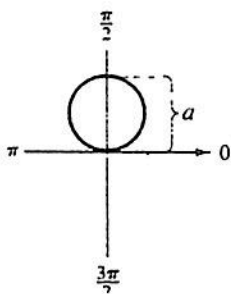


$r = a \sin n\theta$
Rose curve

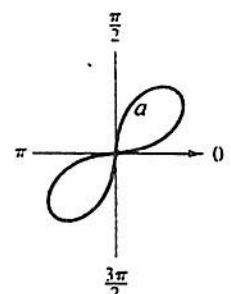
Circles and Lemniscates



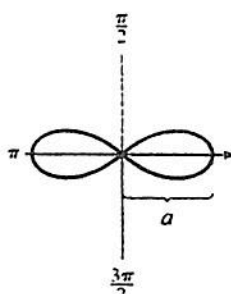
$r = a \cos \theta$
Circle



$r = a \sin \theta$
Circle



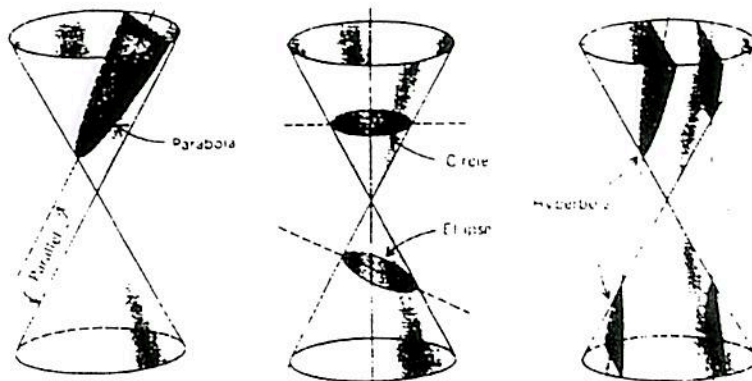
$r^2 = a^2 \sin 2\theta$
Lemniscate



$r^2 = a^2 \cos 2\theta$
Lemniscate

Conics Reference Sheet

A conic section is any figure that can be formed by slicing a double cone.



* distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

* midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Parabola

A parabola is the set of all points in a plane that are the same distance from a given point called the focus and a given line called the directrix.

form of equation: $y = a(x - h)^2 + k$

axis of symmetry: $x = h$

vertex: (h, k)

focus: $\left(h, k + \frac{1}{4a} \right)$

directrix: $y = k - \frac{1}{4a}$

direction of opening: upward if $a > 0$
downward if $a < 0$

length of latus rectum $\left| \frac{1}{a} \right|$ units

$x = a(y - k)^2 + h$

$y = k$

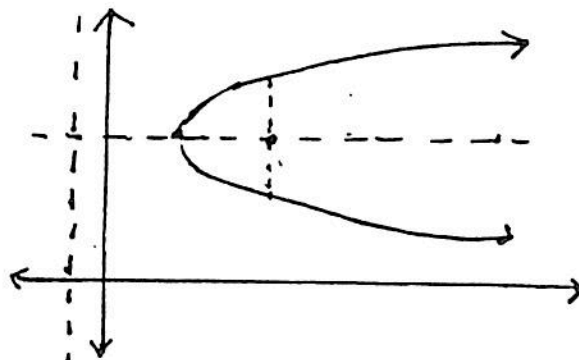
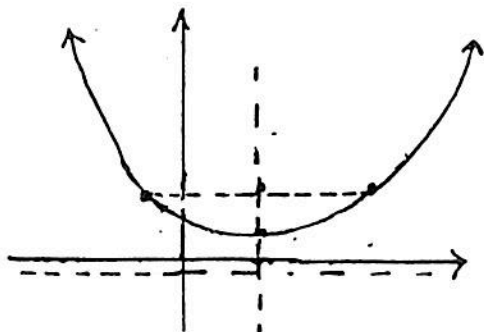
(h, k)

$\left(h + \frac{1}{4a}, k \right)$

$x = h - \frac{1}{4a}$

right if $a > 0$
left if $a < 0$

$\left| \frac{1}{a} \right|$ units

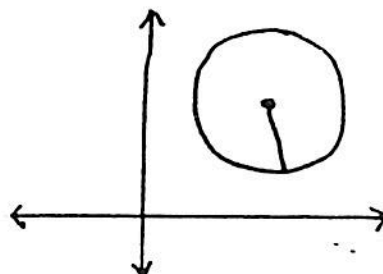


Circle

A circle is the set of points in a plane that are equidistant from a given point in the plane, called the center. Any segment whose endpoints are the center of the circle and a point on the circle is a radius of the circle.

The equation of a circle with center (h, k) and radius r units is:

$$(x - h)^2 + (y - k)^2 = r^2$$



Ellipse

An ellipse is the set of all points in a plane such that the sum of the distances from 2 fixed points (foci) is constant.

Standard form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (a > b)$

center: (h, k)
major axis: horizontal
length = $2a$

minor axis: vertical
length = $2b$

foci: on horizontal axis
(c = dist. from ctr to focus) $c = \sqrt{a^2 - b^2}$

vertices: $(h \pm a, k)$

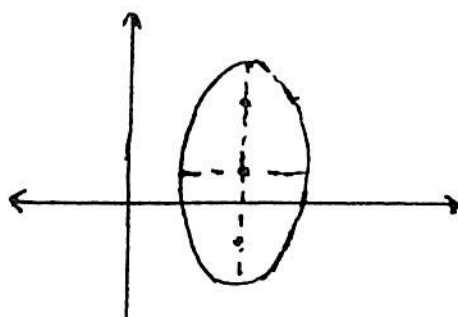
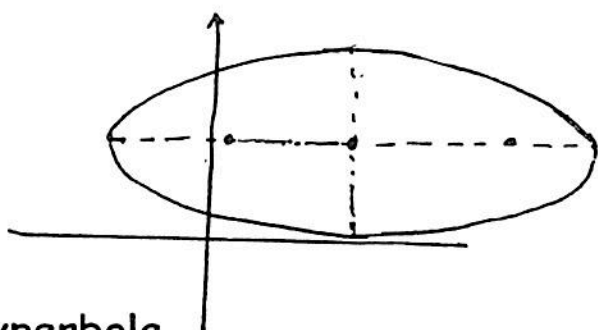
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

(h, k)
vertical
length = $2a$

horizontal
length = $2b$

on vertical axis
 $c = \sqrt{a^2 - b^2}$

$(h, k \pm b)$



Hyperbola

A hyperbola is the set of all points in the plane such that the absolute value of the difference of the distances from any point on the hyperbola to two given points, called the foci, is constant.

Standard form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

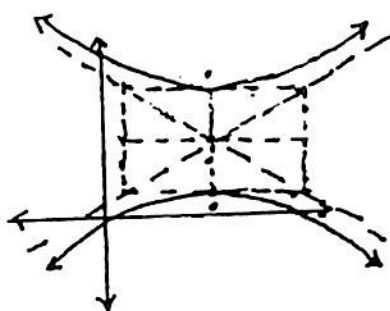
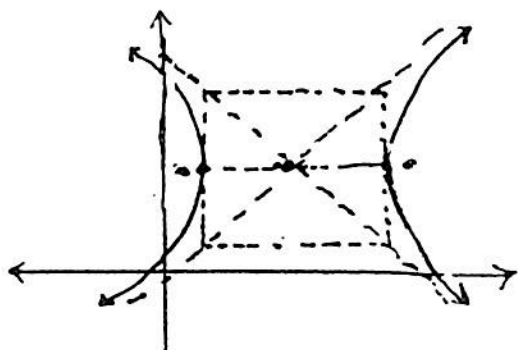
center: (h, k)
transverse axis: horizontal
foci: on transverse axis
(c = dist. from ctr to focus) $c = \sqrt{a^2 + b^2}$

vertices: $(h \pm a, k)$
asymptotes: $(y - k) = \pm \frac{b}{a}(x - h)$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

(h, k)
vertical
on transverse axis
 $c = \sqrt{a^2 + b^2}$

$(h, k \pm a)$
 $(y - k) = \pm \frac{a}{b}(x - h)$



SEQUENCES AND SERIES

Arithmetic Sequence - common difference "d"

$$t_n = t_1 + (n - 1)d$$

$$S_n = \left(\frac{n}{2}\right)(2t_1 + (n - 1)d)$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

Geometric Sequence - common ratio "r"

$$t_n = t_1 r^{n-1}$$

$$S_n = t_1 \frac{1 - r^n}{1 - r}$$

Infinite Geometric Series

$$S = \frac{t_1}{1 - r}, |r| < 1$$

Test	Series	Condition(s) of Convergence	Conditions of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	Although this test cannot show convergence, it does show divergence.
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n \rightarrow \infty} b_n = L$		Sum: $S = b_1 - L$
Geometric Series	$\sum_{n=1}^{\infty} ar^n$	$ r < 1$	$ r \geq 1$	Sum: $S = \frac{a}{1-r}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$		Remainder: $ R_N \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x) dx$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$	Test is inconclusive if $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } > 1$	Test is inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = 1$
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

STRATEGIES FOR SEQUENCES AND SERIES

If a sequence $\{a_n\}$ has a limit L , then the sequence is said to converge to L .

If a series converges, then a_n approaches 0 as n increases (goes to ∞). a_n must approach 0 as n increases to ∞ for a series to converge. The converse is not necessarily true.

The Alternating Series Test is easy to use since it requires that (1) only consecutive terms must be compared $|a_{n+1}| \leq |a_n|$ and (2) the limit of the n th term is zero. Both conditions must hold.

For the Comparison Test to work, it is only necessary for the terms to be eventually less than the convergent terms (or greater than the divergent terms) for all terms after a certain value for n . A finite number of terms which are not less than the corresponding terms in the convergent series will not affect the convergence.

The Comparison Test will only be conclusive if you show that a series is:

1. less than a convergent or
2. greater than a divergent

Proving that a series is greater than a convergent series is inconclusive.

If $p = 1$, the p -series is the harmonic series. Convergence for other p -series can be tested by the Integral Test.

Certain tests are more helpful for certain forms. Terms that can be integrated easily suggest the Integral Test. Factorial notation generally lends itself to the Ratio Test. A sequence that does not converge to zero may suggest the n th term test.

Absolute convergence implies conditional convergence. If $\sum |a_n|$ converges, then $\sum a_n$ must also.

If $\sum a_n$ converges but $\sum |a_n|$ does not, then the series is conditionally convergent.

The alternating harmonic series converges by the Alternating Series Test; yet the harmonic series diverges. This is an example of conditional convergence.

Special types of series such as geometric, p -series, telescoping, or alternating are useful for comparison.

CONVERGENCE/DIVERGENCE

1) CHECK FOR ALTERNATING SERIES

SERIES CONVERGES IFF:

- a) SERIES IS STRICTLY ALTERNATING
- b) EACH TERM GETS SMALLER AND SMALLER (SERIES IS STRICTLY DECREASING)
- c) THE LAST TERM IN THE SERIES APPROACHES ZERO
 $(\lim_{n \rightarrow \infty} a_n = 0)$

IF SERIES IS NOT ALTERNATING:

RULE TO REMEMBER:

A MULTIPLE OF A CONVERGENT SERIES, CONVERGES; WHILE A MULTIPLE OF A DIVERGENT SERIES, DIVERGES.

1) CHECK Nth TERM TEST

SERIES DIVERGES IF:

THE LAST TERM IN THE SERIES DOESN'T APPROACH ZERO
 $(\lim_{n \rightarrow \infty} a_n \neq 0)$

2) CHECK FOR GEOMETRIC SERIES

series converges if: $|r| < 1$

series diverges if: $|r| \geq 1$

note: sum of a geometric series (if $|r| < 1$) is: $S = \frac{a}{1-r}$

3) CHECK FOR P SERIES.

if $a_n = \frac{1}{n^p}$, then:

series converges if $p > 1$

series diverges if $p \leq 1$

4) TRY Nth ROOT TEST.

$$\text{Let } \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = p$$

series converges if: $p < 1$

series diverges if: $p > 1$

try another test if: $p = 1$

$$\text{note: } \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x \text{ and } \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

5) TRY RATIO TEST.

(ALWAYS USE RATIO TEST WHEN YOU HAVE A FACTORIAL!)

$$\text{Let } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = P$$

series converges if: $p < 1$

series diverges if: $p > 1$

try another test if: $p = 1$

$$\text{note: } \lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$$

6) TRY COMPARISON TEST

*series converges if $a_n \leq c_n$ for all $n > n_0$. n_0 is any n .
(c_n is a known convergent series)*

*series diverges if $a_n \geq d_n$ for all $n > n_0$. n_0 is any n .
(d_n is a known divergent series)*

7) TRY THE INTEGRAL TEST

series converges if $\lim_{b \rightarrow \infty} \int_1^b a_n$ converges

series diverges if $\lim_{b \rightarrow \infty} \int_1^b a_n$ diverges

8) TRY THE LIMIT COMPARISON TEST

if $\lim_{n \rightarrow \infty} \frac{a_n}{c_n} < \infty$, then a_n converges

(c_n is a known convergent series)

if $\lim_{n \rightarrow \infty} \frac{a_n}{d_n} > 0$, then a_n diverges

(d_n is a known divergent series)

Lines and Linear Approximations

point-slope form $y - y_1 = m(x - x_1)$
or
 $y_2 - y_1 = m(x_2 - x_1)$

slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

derivative - page 108 $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

alternative form of
derivative - page 110 $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$

tangent line - page 231 $y - f(c) = f'(c)(x - c)$
 $y = f(c) + f'(c)(x - c)$

differential - page 235 $f(x + \Delta x) = f(x) + f'(x)dx$

Taylor Polynomial - page 598

$$P_n(x) = f(c) + f'(c)(x - c) + f''(c)\frac{(x - c)^2}{2!} + \dots + f^n(c)\frac{(x - c)^n}{n!}$$

$$P_1(x) = f(c) + f'(c)(x - c)$$

Euler's Method

$$y_{\text{new}} = y_{\text{old}} + y'_{\text{old}} \Delta x$$
$$f(x_{n+1}) = f(x_n) + f'(x_n)\Delta x$$

