MVT, IVT, EVT

1. Let g be a continuous function of the closed interval $-1 \le x \le 3$. If $g(-1) = -10$ and $g(3) = 6$, which of the following are guaranteed?

   (A) $g(0) = 0$
   (B) $g'(c) = 0$ for some c in the interval $-1 < x < 3$
   (C) $g'(c) = 4$ for some c in the interval $-1 < x < 3$
   (D) $g(c) = 4$ for some c in the interval $-1 < x < 3$
   (E) $-10 \le g(c) \le 6$ for all x between -1 and 3

2. The function h is continuous on the closed interval $[-2, 5]$ and differentiable on the open interval $(-2, 5)$. If $f(-2) = 3$ and $f(5) = -11$, which of the following statements could be false?

   (A) There exists a c on the interval $(-2, 5)$ such that $f'(c) = 0$.
   (B) There is an absolute maximum value on the interval $[-2, 5]$.
   (C) There exists a c on the interval $[-2, 5]$ such that $f(c) \le f(x)$ for all x on the interval $[-2, 5]$.
   (D) There exists a c on the interval $(-2, 5)$ such that $f(c) = 0$.
   (E) There exists a c on the interval $(-2, 5)$ such that $f'(c) = -2$.

3. Let g be a function that is differentiable over the interval $(2, 9)$. Given $g(3) = 5$, $g(6) = -2$, and $g(8) = 5$, which of the following must be true?

   I. g has at least one horizontal tangent line.
   II. g has at least 2 zeros.
   III. For some c in the interval $(3, 6)$, $f'(c) = -\frac{7}{3}$.

   (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

4. If $f(x) = x^3 + 1$, then there exists a number c in the interval $(0, 1)$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

(A) $-\sqrt{\frac{1}{3}}$ (B) 0 (C) $\sqrt{\frac{1}{3}}$ (D) 1 (E) 2

5. $h(x)$ is a differentiable function that contains the points $(2, -5)$ and $(5, 4)$. Which of the following must be true?

(A) $h(x)$ is increasing over the interval $(2, 5)$.

(B) $h(x)$ intercepts the $x-$axis at $\frac{11}{3}$.

(C) $h'(c) = 0$ for some c in the interval $(-5, 4)$.

(D) $h'(c) = 3$ for some c in $(2, 5)$

(E) $h'(c) = 3$ for all x in $(2, 5)$

6. The Mean Value Theorem may be applied to which of the following functions over the interval named?

(A) $f(x) = \tan x$ over $[0, \pi]$

(B) $f(x) = |x|$ over $[-1, 1]$

(C) $f(x) = \frac{1}{x}$ over $[1, 2]$

(D) $f(x) = \frac{\sin x}{x}$ over $[-\pi, \pi]$

(E) $f(x) = [x]$ over $[1, 3]$