AP CALCULUS AB
FORMULA LIST

Definition of e: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Definition of absolute value: $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

Definition of the derivative:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Alternative form:

$$f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

Definition of continuity: f is continuous at $x = c$ if and only if

1) $f(c)$ is defined;
2) $\lim_{x\to c} f(x)$ exists;
3) $\lim_{x\to c} f(x) = f(c)$.

Average rate of change of $f(x)$ on $[a, b] = \dfrac{f(b)-f(a)}{b-a}$

Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

Rolle's Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there is at least one number c on (a, b) such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

$\cos^2 x + \sin^2 x = 1$ $\quad$ $1+\tan^2 x = \sec^2 x$ $\quad$ $1+\cot^2 x = \csc^2 x$

$\sin(2x) = 2\sin x\cos x$

$$\cos(2x) = \begin{cases} \cos^2 x - \sin^2 x \\ 1-2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$\frac{d}{dx}[c] = 0$ $\quad$ $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$ $\quad$ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$

$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)\cdot g'(x)$

$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx}$

$\frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$

$\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx}$

$\frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx}$

$\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx}$

$\frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$

$\frac{d}{dx}[\ln u] = \frac{1}{u} \frac{du}{dx}$

$\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a} \frac{du}{dx}$

$\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

$\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$

$\left(f^{-1}\right)'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$

<u>Definition of a Critical Number</u>:
Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of *f*.

<u>First Derivative Test</u>:
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at $x = c$, then $f(c)$ can be classified:

1) If $f'(x)$ changes from negative to positive at $x = c$, then $(c, f(c))$ is a **relative minimum** of *f*.

2) If $f'(x)$ changes from positive to negative at $x = c$, then $(c, f(c))$ is a **relative maximum** of *f*.

<u>Second Derivative Test</u>:
Let f be a function such that the second derivative of f exists on an open interval containing c.

1) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ is a **relative minimum** of *f*.

2) If $f'(c) = 0$ and $f''(c) < 0$, then $(c, f(c))$ is a **relative maximum** of *f*.

<u>Definition of Concavity</u>:
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

<u>Test for Concavity</u>:
Let f be a function whose second derivative exists on an open interval I.

1) If $f''(x) > 0$ for all x in the interval *I*, then the graph of f is **concave upward** in I.

2) If $f''(x) < 0$ for all x in the interval *I*, then the graph of f is **concave downward** in I.

<u>Definition of an Inflection Point</u>:
A function f has an inflection point at $(c, f(c))$

1) if $f''(c) = 0$ or $f''(c)$ does not exist <u>and</u>

2) if f'' changes sign from positive to negative or negative to positive at $x = c$

<u>OR</u> if $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x = c$.

Definition of a definite integral: $\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k)\cdot(\Delta x_k) = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot(\Delta x_k)$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int \cos u\,du = \sin u + C$

$\int \sin u\,du = -\cos u + C$

$\int \sec^2 u\,du = \tan u + C$

$\int \csc^2 u\,du = -\cot u + C$

$\int \sec u \tan u\,du = \sec u + C$

$\int \csc u \cot u\,du = -\csc u + C$

$\int \frac{1}{u}\,du = \ln|u| + C$

$\int \tan u\,du = -\ln|\cos u| + C$

$\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\int e^u\,du = e^u + C$

$\int a^u\,du = \frac{a^u}{\ln a} + C$

First Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Chain Rule Version: $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$

Average value of $f(x)$ on $[a, b]$: $f_{AVE} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}[\arccos u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}[\operatorname{arc\,cot} u] = -\frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}[\operatorname{arc\,sec} u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$\frac{d}{dx}[\operatorname{arc\,csc} u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arc\,sec}\frac{|u|}{a} + C$

Volume by cross sections taken perpendicular to the x-axis: $V = \int_a^b A(x)dx$,

where $A(x) =$ area of each cross section

Volume around a horizontal axis by discs: $V = \pi \int_a^b [r(x)]^2 dx$

Volume around a horizontal axis by washers: $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)dx$

Volume around a vertical axis by shells: $V = 2\pi \int_a^b r(x)\, h(x)\, dx$

If an object moves along a straight line with position function $s(t)$, then its

Velocity is $v(t) = s'(t)$ Speed = $|v(t)|$

Acceleration is $a(t) = v'(t) = s''(t)$

Displacement (change in position) from $x = a$ to $x = b$ is Displacement = $\int_a^b v(t)\, dt$

Total Distance traveled from $x = a$ to $x = b$ is Total Distance = $\int_a^b |v(t)\, dt|$

or Total Distance = $\left|\int_a^c v(t)\, dt\right| + \left|\int_c^b v(t)\, dt\right|$, where $v(t)$ changes sign at $x = c$.

The speed of the object is **increasing** when its velocity and acceleration have the same sign.
The speed of the object is **decreasing** when its velocity and acceleration have opposite signs.
