AP CALCULUS BC
FORMULA LIST

Definition of e: $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

Definition of absolute value: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Alternative form:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Definition of continuity: f is continuous at $x = c$ if and only if

1) $f(c)$ is defined;
2) $\lim_{x \to c} f(x)$ exists;
3) $\lim_{x \to c} f(x) = f(c)$.

Average rate of change of $f(x)$ on $[a, b] = \dfrac{f(b) - f(a)}{b - a}$

Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

Rolle's Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there is at least one number c on (a, b) such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

$\cos^2 x + \sin^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$

$$\cos(2x) = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$$\left.\begin{aligned} \cos^2 x &= \frac{1 + \cos(2x)}{2} \\ \sin^2 x &= \frac{1 - \cos(2x)}{2} \end{aligned}\right\} \text{ needed second semester in BC}$$

$\dfrac{d}{dx}[c] = 0$

$\dfrac{d}{dx}\left[x^n\right] = nx^{n-1}$

$\dfrac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$

$\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$

$\dfrac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$

$$\frac{d}{dx}[\sin u] = \cos u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\cos u] = -\sin u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\cot u] = -\csc^2 u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\csc u] = -\csc u \cot u \, \frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \, \frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a} \, \frac{du}{dx}$$

$$\frac{d}{dx}[e^u] = e^u \, \frac{du}{dx}$$

$$\frac{d}{dx}[a^u] = a^u \ln a \, \frac{du}{dx}$$

$$\left(f^{-1}\right)'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$$

Definition of a Critical Number:

Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of f.

Definition of Increasing and Decreasing Functions

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval,
$x_1 < x_2$ implies $f(x)_1 < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval,
$x_1 < x_2$ implies $f(x)_1 > f(x_2)$.

Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a,b).

1) If $f'(x) > 0$ for all x in (a,b), then f is increasing on $[a, b]$.

2) If $f'(x) < 0$ for all x in (a,b), then f is decreasing on $[a, b]$.

1) If $f'(x) = 0$ for all x in (a,b), then f is constant on $[a, b]$.

First Derivative Test:

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at $x = c$, then $(c, f(c))$ can be classified as follows:

1) If $f'(x)$ changes from negative to positive at $x = c$, then $(c, f(c))$ is a **relative or local minimum** of f.

2) If $f'(x)$ changes from positive to negative at $x = c$, then $(c, f(c))$ is a **relative or local maximum** of f.

Second Derivative Test:

Let f be a function such that the second derivative of f exists on an open interval containing c.

1) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ is a **relative or local minimum** of f.

2) If $f'(c) = 0$ and $f''(c) < 0$, then $(c, f(c))$ is a **relative or local maximum** of f.

<u>Definition of Concavity</u>:
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

<u>Test for Concavity</u>:
Let f be a function whose second derivative exists on an open interval I.
1) If $f''(x) > 0$ for all x in the interval I, then the graph of f is **concave upward** in I.
2) If $f''(x) < 0$ for all x in the interval I, then the graph of f is **concave downward** in I.

<u>Definition of an Inflection Point</u>:
A function f has an inflection point at $(c,\ f(c))$
1) if $f''(c) = 0$ or $f''(c)$ does not exist <u>and</u>
2) if f'' changes sign from positive to negative or negative to positive at $x = c$
<u>OR</u> if $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x = c$.

Definition of a definite integral: $\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k)\cdot(\Delta x_k) = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot(\Delta x_k)$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int \cos u\,du = \sin u + C$

$\int \sin u\,du = -\cos u + C$

$\int \sec^2 u\,du = \tan u + C$

$\int \csc^2 u\,du = -\cot u + C$

$\int \sec u \tan u\,du = \sec u + C$

$\int \csc u \cot u\,du = -\csc u + C$

$\int \frac{1}{u}\,du = \ln|u| + C$

$\int \tan u\,du = -\ln|\cos u| + C$

$\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\int e^u\,du = e^u + C$

$\int a^u\,du = \frac{a^u}{\ln a} + C$

First Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Chain Rule Version: $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$

Average value of $f(x)$ on $[a, b]$: $f_{AVE} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arccos u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arccot} u] = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arccsc} u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Volume by cross sections taken perpendicular to the *x*-axis: $V = \int_a^b A(x)dx$,

where $A(x) =$ area of each cross section

Volume around a horizontal axis by discs: $V = \pi\int_a^b \left(r(x)\right)^2 dx$

Volume around a horizontal axis by washers: $V = \pi\int_a^b \left(\left(R(x)\right)^2 - \left(r(x)\right)^2\right)dx$

Volume around a vertical axis by shells: $V = 2\pi\int_a^b r(x)h(x)dx$ (not required on the AP test)

If an object moves along a straight line with position function $s(t)$, then its

Velocity is $v(t) = s'(t)$

Speed = $|v(t)|$

Acceleration is $a(t) = v'(t) = s''(t)$

Displacement (change in position) from $x = a$ to $x = b$ is Displacement = $\int_a^b v(t)\,dt$

Total Distance traveled from $x = a$ to $x = b$ is Total Distance = $\int_a^b |v(t)\,dt|$

or Total Distance = $\left|\int_a^c v(t)\,dt\right| + \left|\int_c^b v(t)\,dt\right|$, where $v(t)$ changes sign at $x = c$.

The speed of the object is **increasing** when its velocity and acceleration have the same sign.

The speed of the object is **decreasing** when its velocity and acceleration have opposite signs.

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Differential equation for logistic growth: $\frac{dP}{dt} = kP(L-P)$, where $L = \lim_{t\to\infty} P(t)$. The population is growing fastest when $P = \frac{1}{2}L$ because this is when $\frac{d^2P}{dt^2}$ changes from positive to negative (so that this is where the inflection point of the solution curve occurs).

Integration by parts: $\int u\,dv = uv - \int v\,du$	Length of arc for functions: $s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

If an object moves along a curve, its

Position vector = $\langle x(t),\ y(t)\rangle$

Velocity vector = $\langle x'(t),\ y'(t)\rangle$

Acceleration vector = $\langle x''(t),\ y''(t)\rangle$

Speed (or magnitude of velocity vector) = $|v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Distance traveled from $t = a$ to $t = b$ (or length of arc) is $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

In polar curves, $x = r\cos\theta$ and $y = r\sin\theta$

Slope of polar curve: $\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

Area within a polar curve: $A = \frac{1}{2}\int_a^b r^2\,d\theta$

If f has n derivatives at $x = c$, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the **nth Taylor polynomial for *f* centered at *c*.**

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the **Taylor series for *f* centered at *c*.**

Lagrange Error Bound for a Taylor Polynomial (or Taylor's Theorem Remainder):

Taylor's Theorem: If a function f is differentiable through order $n+1$ in an interval containing c, then for each x in the interval, there exists a number z between x and c such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x)$$

where $R_n(x) = \dfrac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$.

The remainder represents the difference between the function and the polynomial. That is,

$$\left|R_n(x)\right| = \left|f(x) - P_n(x)\right| .$$

One useful consequence of Taylor's Theorem is that $R_n(x) = \dfrac{|x-c|^{n+1}}{(n+1)!}\max\left|f^{(n+1)}(z)\right|$, where $\max\left|f^{(n+1)}(z)\right|$ is the maximum value of $f^{(n+1)}(z)$ between x and c. This gives us a **bound** for the error. It does not give us the exact value of the error. The bound is called **Lagrange's form of the remainder** or the **Lagrange error bound**.

Alternating Series Remainder:

If a series has terms that alternate, decrease in absolute value, and have a limit of 0 (so that the series converges by the Alternating Series Test), then the absolute value of the remainder R_n involved in approximating the sum S by S_n is less than the first neglected term. That is,

$$\left|R_n\right| = \left|S - S_n\right| < a_{n+1} .$$

Maclaurin series that you must know:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$$