Ch. 6 Diff Equ. W/S

Ex 1: $dy/dx = \frac{3x^2}{e^{2y}}$

$\int e^{2y} dy = \int 3x^2 dx$ $\quad u = 2y$

$du = 2dy$

$dy = \frac{du}{2}$

$\frac{1}{2}\int e^u du = x^3 + C$

$\frac{1}{2} e^{2y} = x^3 + C$

$\ln(e^{2y}) = (2x^3 + C)\ln$

$2y = \ln|2x^3 + C|$

$\boxed{y = \frac{1}{2}\ln|2x^3+C}$ $\quad (0, 1/2)$

$1/2 = \frac{1}{2}\ln C$

$\ln C = 1 \quad C = e$

$\therefore \boxed{y = 1/2 \ln|2x^3 + e|}$

Ex 2: $\frac{dy}{dx} = 2y^2$

$\int \frac{dy}{y^2} = 2\int dx$

$-1/y = 2x + C$

$y = \frac{-1}{2x + C} \quad (1, -1) \rightarrow -1 = \frac{-1}{2+C} \quad C = -1$

$\therefore \boxed{y = -1/(2x-1)}$

Ex 3: $\frac{dy}{dx} = y^2(6-2x)$

$\int \frac{dy}{y^2} = \int (6-2x)dx$

$\frac{-1}{y} = 6x - x^2 + C$

$y = \frac{-1}{6x - x^2 + C} \rightarrow (3, 1/4)$

$\frac{1}{4} = \frac{-1}{9+C} \quad C = -13$

$\therefore \boxed{y = \frac{-1}{\ldots}}$

① $\frac{dy}{dx} = \frac{x-3}{y}$ $(2, -5)$

$\int y\,dy = \int (x-3)\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} - 3x + C \rightarrow \frac{25}{2} = -4 + C$

$C = 33/2$

$$\boxed{\begin{aligned} y^2 &= x^2 - 6x + 33 \\ y &= \sqrt{x^2 - 6x + 33} \end{aligned}}$$

② $\frac{dy}{dx} = 2x\sqrt{y}$ $(2, 25)$

$\int \frac{dy}{\sqrt{y}} = \int 2x\,dx$

$2\sqrt{y} = x^2 + C$

$10 = 4 + C$ $C = 6$

$2\sqrt{y} = x^2 + 6$

$\sqrt{y} = \frac{x^2}{2} + 3$ $\boxed{y = \left(\frac{x^2}{2} + 3\right)^2}$

③. $\frac{dy}{dx} = 4y^2 \sec^2(2x)$ $(\pi/8, 1)$

$\int \frac{dy}{y^2} = \int 4 \sec^2(2x)\,dx$ $u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$\frac{-1}{y} = 2\tan(2x) + C$

$y = \frac{-1}{2\tan(2x) + C} \rightarrow 1 = -\frac{1}{2\tan(\pi/4) + C}$

$1 = \frac{-1}{2 + C}$ $C = -3$

$$\boxed{y = \frac{-1}{2\tan(2x) - 3}}$$

④ $xy\frac{dy}{dx} = \ln x \qquad (1,2)$

$\int y\,dy = \int \frac{\ln x\,dx}{x} \qquad u = \ln x \quad du = \frac{1}{x}dx \quad dx = x\,du$

$\int y\,dy = \int u\,du$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$

$y^2 = (\ln x)^2 + C \rightarrow 4 = (\ln 1)^2 + C \quad C = 4$

$\boxed{y^2 = (\ln x)^2 + 4}$

⑤ $\frac{dy}{dx} = 2x\sec y \qquad (2, -\pi/2)$

$\frac{dy}{\sec y} = 2x\,dx$

$\int \cos y\,dy = \int 2x\,dx$

$\sin y = x^2 + C \rightarrow \sin(-\pi/2) = 4 + C \rightarrow -1 = 4 + C \quad C = -5$

$\sin y = x^2 - 5 \qquad \boxed{y = \arcsin(x^2 - 5)}$

⑥ $\frac{dy}{dx} - \cancel{xe^y} = 2e^y + xe^y \quad (0,0)$

$\frac{dy}{dx} = e^y(x+2)$

$\int \frac{dy}{e^y} = \int (x+2)\,dx$

$\int e^{-y}\,dy = \int (x+2)\,dx$

$-e^{-y} = \frac{x^2}{2} + 2x + C$

$-\ln e^{-y} = \ln\left(\frac{x^2}{2} + 2x + C\right)$

$y = \ln\left(\frac{x^2}{2} + 2x + C\right) \rightarrow 0 = \ln(C)$

$C = 1$

$\boxed{y = \ln\left(\frac{x^2}{2} + 2x + 1\right)}$

⑦ $\frac{dy}{dx} = 2xy^3\sin(x^2) \qquad (0,-1)$

$\int \frac{dy}{y^3} = \int 2x\sin(x^2)\,dx \qquad u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$-\frac{1}{2y^2} = \int \sin u\,du$

$-\frac{1}{2y^2} = -\cos(x^2) + C$

$-\frac{1}{2} = -\cos 0 + C$

$-\frac{1}{2} = -1 + C \quad C = \frac{1}{2}$

$\frac{-1}{2y^2} = -\cos(x^2) + \frac{1}{2}$

$\boxed{y^2 = \frac{1}{2\cos x^2 + 1}}$

⑧ $\int y^2 dy = \int dx \quad (0,4)$

$\frac{y^3}{3} = x + C \rightarrow \frac{64}{3} = C$

$\frac{y^3}{3} = x + \frac{64}{3}$

$\boxed{\begin{aligned} y^3 &= 3x + 64 \\ y &= \sqrt[3]{3x+64} \end{aligned}}$

⑨ $\frac{dy}{dx} = \frac{2x}{y^2} \quad (0,3)$

$\int y^2 dy = \int 2x\,dx$

$\frac{y^3}{3} = x^2 + C \rightarrow 9 = C$

$\frac{y^3}{3} = x^2 + 9$

$\boxed{\begin{aligned} y^3 &= 3x^2 + 27 \\ y &= \sqrt[3]{3x^2+27} \end{aligned}}$

W/S 2

① $\int \frac{dy}{y} = \int 6x^2 dx$

$\ln y = 2x^3 + C \quad (0,4)$

$C = \ln 4$

$\ln y = 2x^3 + \ln 4$

$y = e^{2x^3 + \ln 4}$

$y = e^{2x^3} \cdot e^{\ln 4}$

$\boxed{y = 4e^{2x^3}}$

② $\int \sqrt{y}\, dy = \int (1+x)dx \quad (2,9)$

$\frac{2}{3} y^{3/2} = x + \frac{x^2}{2} + C$

$18 = 2 + 2 + C, \ C = 14$

$\frac{2}{3} y^{3/2} = \frac{1}{2}x^2 + x + 14$

$\boxed{y^{3/2} = 3x^2 + \frac{2}{3}x + 21}$

③ $\int \frac{dy}{(y-2)^2} = \int \cos(3x)\,dx \quad (\pi/2, 3)$

$\frac{-1}{y-2} = \frac{1}{3}\sin(3x) + C$

$-1 = \frac{-1}{3} + C \quad C = -2/3$

$\frac{-1}{y-2} = \frac{1}{3}\sin(3x) - \frac{2}{3}$

$\frac{-3}{y-2} = \sin(3x) - 2$

$y - 2 = \frac{-3}{\sin(3x)-2}$

$\boxed{y = 2 - \frac{3}{\sin(3x)-2}}$

W/S 2 - DE

(8) $\int \frac{dy}{y} = \int \cos x\, dx \quad (0,3)$

$\ln|y| = \sin x + C$

$\ln 3 = 0 + C$

$C = \ln 3$

$\ln|y| = \sin x + \ln 3$

$|y| = e^{\sin x + \ln 3}$

$|y| = e^{\sin x} \cdot e^{\ln 3}$

$|y| = 3e^{\sin x}$

$\therefore \boxed{y = 3e^{\sin x}}$

(9) $\int \frac{dy}{y^2} = \int 2\, dx \quad (1,-1) \rightarrow (2, \overset{?}{y})$

$\frac{-1}{y} = 2x + C \quad 1 = 2 + C \quad C = -1$

$\frac{-1}{y} = 2x - 1$

$\boxed{y = \frac{-1}{2x-1} \rightarrow y(2) = -\frac{1}{3}}$ (B)

(10) $\int \frac{dy}{y^3} = \int x\, dx \quad (2,-1)$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-1/2 = 2 + C \quad C = -\frac{5}{2}$

$\frac{-1}{2y^2} = \frac{x^2-5}{2}$

$y^2 = \frac{-1}{x^2-5}$

$y = \sqrt{\frac{-1}{x^2-5}}$

Domain: $-\sqrt{5} < x < \sqrt{5}$

(11) $\int \frac{dy}{y+5} = \int \frac{1}{x}\, dx \quad -3, 1$

$\ln|y+5| = \ln|x| + C$

$\ln 6 = \ln 3 + C$

$C = \ln 6 - \ln 3$

$C = \ln \frac{6}{3} = \ln 2$

$\ln|y+5| = \ln|x| + \ln 2$

$|y+5| = e^{\ln|x|} \cdot e^{\ln 2}$

$|y+5| = 2|x|$

$y = \pm(2|x| - 5)$

Domain: $\mathbb{R}, x \neq 0$

W/S: Applications of DE

Calc ① $y' = K(y - T_0) \Rightarrow y = Ce^{Kt}$

$(0, 450)$ $(30, 200)$

$\frac{dy}{dt} = K(y - 70)$

$\int \frac{dy}{y-70} = \int K\,dt$

$\ln|y-70| = Kt + C$ ($y > 70$, so remove $|\ |$)

$\ln(y-70) = Kt + C$

$y - 70 = e^{Kt+C}$

$y - 70 = e^{Kt} \cdot e^{C}$

$y - 70 = Ce^{Kt}$ use $(0, 450)$ to find C

$380 = Ce^{0}$ $C = \underline{380}$

$y = 380e^{Kt}$ use $(30, 200)$ to find K

$200 = 380e^{30K}$

$e^{30K} = \frac{10}{19}$

$30K = \ln\left(\frac{10}{19}\right)$

$K = \frac{1}{30}\ln\left(\frac{10}{19}\right) \approx -.021$

$\therefore y = 70 + 380e^{-.021t}$ is the model to use to solve $y = 100$ @ $t = 120$ minutes

② $y = Ce^{Kt}$ & $y' = Kt$ use $(0, 2500)$ Initial to find C

$2500 = Ce^{0}$ $C = 2500$

★ $y = 2500e^{Kt}$ use $(5, 3600)$ to find K

$3600 = 2500e^{5K}$

$1.44 = e^{5K}$

$\ln 1.44 = 5K$ $K \approx .073$

$\boxed{\therefore y = 2500e^{.073t}}$

③ $y = Ce^{kt}$ use $(0, 36)$ to find C

$36 = Ce^{0}$ $C = 36$

$y = 36e^{kt}$ use $(1, 35)$ to find K

$35 = 36e^{K}$

$\frac{35}{36} = e^{K}$

$\ln\left(\frac{35}{36}\right) = K \approx -.028$

$\therefore y = 36e^{-.028t}$

④ $\frac{dy}{dt} = K(50-y)$

$\int \frac{dy}{50-y} = \int K\,dt$

$\ln|50-y| = Kt + C$ use $(0,0)$ to find C

$\ln 50 = C$

$\ln|50-y| = Kt + \ln 50$

$|50-y| = e^{Kt} \cdot e^{\ln 50}$

$y \leq 50$ so ok to drop $| \; |$ $50 - y = 50e^{Kt}$

$y = 50 - 50e^{Kt}$ use $(.5, 20)$ to find K

$50e^{.5K} = 30$

$e^{.5K} = \frac{3}{5}$

$.5K = \ln 3/5$

$K \approx -1.022$

$\therefore y = 50 - 50e^{-1.022t}$ $y(2) \approx 44$ verbs

$y = 49$ @ $t \approx 3.828$ hrs

5) (a) $P(t) = 800 - 500e^{Kt}$

(b) $700 = 800 - 500e^{2K}$

$e^{2K} = \frac{1}{5}$

$K = \frac{\ln(.2)}{2} \approx -.805$

(c) $P(t) = 800 - 500e^{-.805t}$

$\therefore \lim_{t\to\infty} P(t) = 800$

6) $\frac{dy}{dx} = xy^2 \quad (1,1)$

$\int \frac{dy}{y^2} = \int x\,dx$

$-\frac{1}{y} = \frac{x^2}{2} + C$

$-1 = \frac{1}{2} + C \quad C = -\frac{3}{2}$

$-\frac{1}{y} = \frac{x^2-3}{2}$

$y = \frac{-2}{x^2-3}$

7) $\frac{dy}{dx} = 2x + 2xy \quad (0,3)$

$\frac{dy}{dx} = 2x(1+y)$

$\int \frac{dy}{1+y} = \int 2x\,dx$

$\ln|1+y| = x^2 + C$

$\ln|1+y| = x^2 + \ln 4$

$|1+y| = e^{x^2} \cdot e^{\ln 4}$

$|1+y| = 4e^{x^2}$

$y = \pm(4e^{x^2} - 1)$

8) $\frac{dy}{dx} = \frac{3x}{y} \quad (1,3)$

$\int y\,dy = \int 3x\,dx$

$\frac{y^2}{2} = \frac{3}{2}x^2 + C$

$\frac{9}{2} = \frac{3}{2} + C \quad C = 3$

$\frac{y^2}{2} = \frac{3}{2}x^2 + 3$

$y^2 = 3x^2 + 6$

$y = \pm\sqrt{3x^2+6}$

9) $2x\frac{dy}{dx} = \ln(x^2) \quad (1,3)$

$\int dy = \int \frac{\cancel{2}\ln(x)}{2x}\,dx \qquad u = \ln x$

$du = \frac{1}{x}dx$

$\int dy = \int u\,du \qquad dx = x\,du$

$y = \frac{(\ln(x))^2}{2} + C$

$3 = \frac{(\ln 1)^2}{2} + C$

$C = 3$

$\boxed{y = \frac{(\ln(x))^2}{2} + 3}$