8.7 L'Hoptial's Rule & Indeterminate Forms

Analyze $\lim_{x \to -1} \frac{2x^2 - 2}{x + 1}$

We know that x is undefined at x = -1.

Previously we learned some algebraic "tricks" to figure these out…but what if they don't work?

$\lim_{x \to a} \frac{f(x)}{g(x)}$

L'Hopital's Rule:

Functions f and g are differentiable and g'(x) ≠ 0 near a, but could be zero at a. Suppose that:

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

Or that

$$\lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Examples:

1) $\lim_{x \to 1} \frac{\ln x}{x - 1}$

2) $\lim_{x \to \infty} \frac{e^x}{x^2}$

3) A. $\lim_{h \to 0} \frac{e^h - 1}{2h}$

B. $\lim_{x \to 0} \frac{x - sinx}{x^3}$

Sometimes you must rewrite the problem to fit the format to use L'Hopital's rule.

4) $\lim_{x \to 0^+} x \ln x$

5) $\lim_{x \to \frac{\pi}{2}^-} \sec x - \tan x$

If you have $\lim_{x \to a} \left[f(x) \right]^{g(x)}$ you could get 0^0, ∞^0, 1^∞

6) $\lim_{x \to 0^+} x^x$