☆ no Calc on #1

D4
CW/HW

Calculus Name ______________________

Cross Sectional Volume

Write a formula for the area of:

a. Square of side s s^2

b. Equilateral triangle of side s $\frac{\sqrt{3}}{4}s^2$

c. Rectangle $LW = bh$

d. Isosceles right triangle with leg s $\frac{s^2}{2}$

e. Semi-circle with radius r $\frac{\pi r^2}{2}$

Example 1: Emanuel the Duck just bought land with a perimeter set by

$y = -\frac{1}{3}x^2 + 1$, $x = 0$ and $y = 0$ 1st quadrant

He plans to build a Biodome, which uses the area described above as a base. The Biodome will be built up so that cross-sections perpendicular to the x-axis will be squares. He wants to know if he will have enough volume in his biodome to have a party with all his friends. He needs 1 cubic mile of space. What is the volume and will he have enough space to entertain his friends?

$A = s^2$

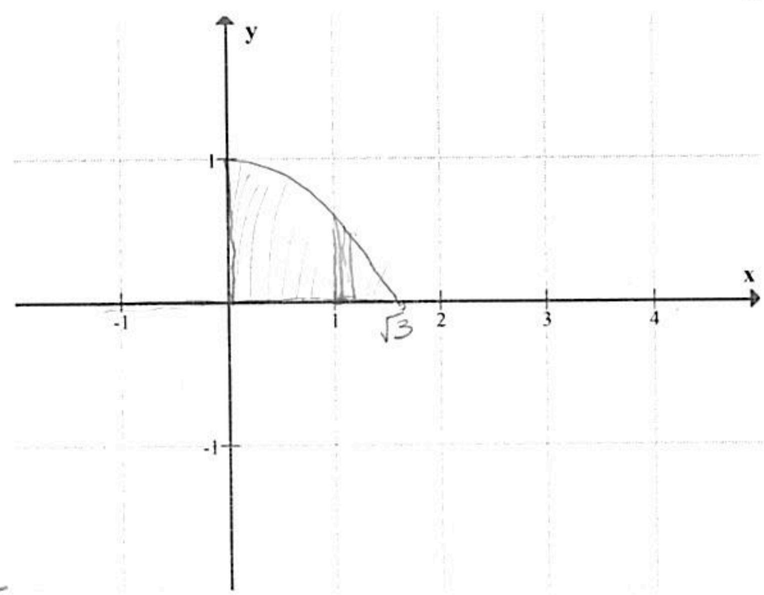
$s = -\frac{1}{3}x^2 + 1$

$s^2 = (-\frac{1}{3}x^2+1)(-\frac{1}{3}x^2+1)$

$-\frac{1}{3}x^2 = -1$

$x^2 = 3$

$x = \pm\sqrt{3}$



$$\int_0^{\sqrt{3}} \left(\frac{1}{9}x^4 - \frac{2}{3}x^2 + 1\right) dx$$

$$\left.\frac{x^5}{45} - \frac{2x^3}{9} + x\right]_0^{\sqrt{3}} = \frac{9\sqrt{3}}{45} - \frac{6\sqrt{3}}{9} + \sqrt{3}$$

$$= \frac{\sqrt{3}}{5} - \frac{2\sqrt{3}}{3} + \sqrt{3} =$$

$$\frac{3\sqrt{3} - 10\sqrt{3} + 15\sqrt{3}}{15} = \boxed{\frac{8\sqrt{3}}{15}}$$

Example 2: Find the volume of the solid whose base is bounded by the equations $y = -x^2 + 4$ and $y = -x - 2$ and whose cross sections taken perpendicular to the x-axis are:

$-x^2+4-(-x-2)=$ $-x^2+x+6$

a. Squares
b. Equilateral Triangles
c. Rectangles of height 1
d. Isosceles Right triangles with l leg in the base
e. Semicircles

$A=s^2$

(a) $s = (-x^2+4)-(-x-2)$

$s^2 = (-x^2+x+6)^2$

$V_s = \int_{-2}^{3} (x^4-2x^3-11x^2+12x+36)\,dx$

$= 104.167$

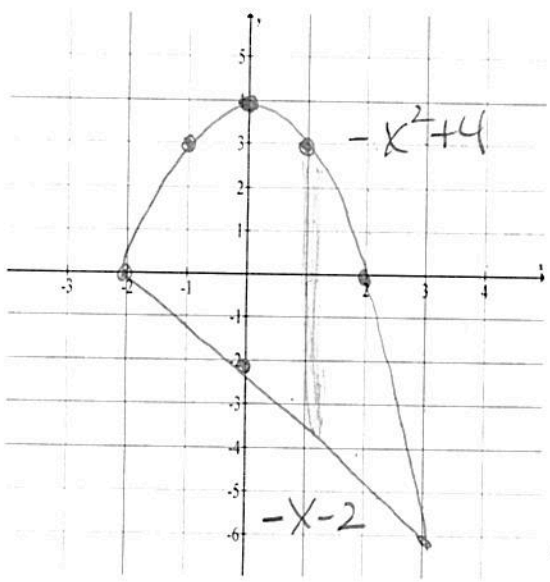
(b) $A = \frac{\sqrt{3}}{4}\int_{-2}^{3}(x^4-2x^3-11x^2+12x+36)\,dx$

$= 45.105$

(c) $V = \int_{-2}^{3}(-x^2+x+6)\,dx = 20.833$

(d) $V = \frac{1}{2}\int_{-2}^{3}(x^4-2x^3-11x^2+12x+36)\,dx =$ 52.083

(e) $V = \frac{\pi}{2}\int_{-2}^{3}\left(\frac{(-x^2+x+6)}{2}\right)^2 = 40.906$



Example 3: Find the volume of the solid whose base is bounded by the equations $y = -x^2 + 3$ and $y = -1$ and whose cross sections taken perpendicular to the y-axis are:

$x^2 = 3-y$

$x = \pm\sqrt{3-y}$

a. Squares
b. Equilateral Triangles
c. Rectangles of height 1
d. Isosceles Right triangles with l leg in the base
e. Semicircles

$A=s^2$

(a) $s = (\sqrt{3-y}) - (-\sqrt{3-y})$

$V = \int_{-1}^{3}(2\sqrt{3-y})^2\,dy = 32$

(b) $A = \frac{\sqrt{3}}{4}s^2 \quad s = 2\sqrt{3-y}$

$V = \frac{\sqrt{3}}{4}\int_{-1}^{3}(2\sqrt{3-y})^2\,dy = 13.856$

(c) $A = bh \quad b = 2\sqrt{3-y}$

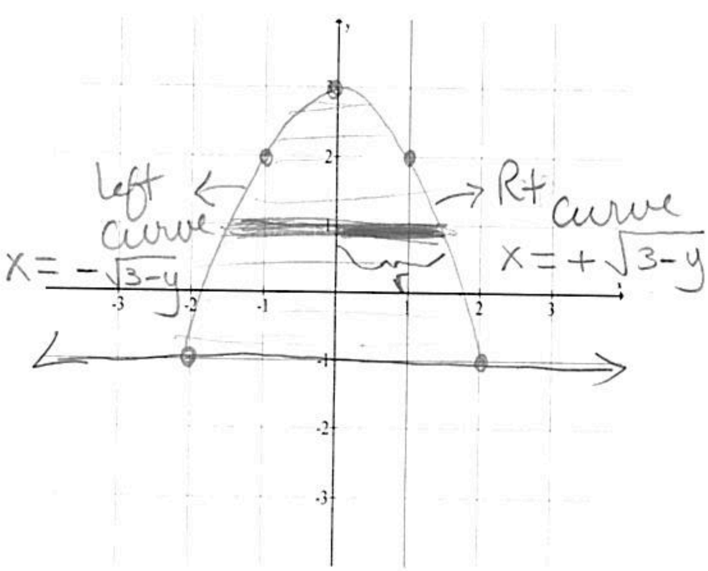
$V = \int_{-1}^{3}(2\sqrt{3-y})\,dy = 10.667$

(d) $A = \frac{s^2}{2}$

$V = \int_{-1}^{3}\frac{(2\sqrt{3-y})^2}{2}\,dy = 16$

(e) $A = \frac{\pi r^2}{2} \quad r = \sqrt{3-y}$

$V = \frac{\pi}{2}\int^{3}(\sqrt{3-y})^2\,dy = 12.566$



D²

Homework

$S = -x^2 + x + 2$

$S^2 = x^4 - 2x^3 - 3x^2 + 4x + 4$

Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$ with the indicated cross sections taken perpendicular to the x-axis.

1. Squares $A = S^2$ $S = (x+1) - (x^2-1)$

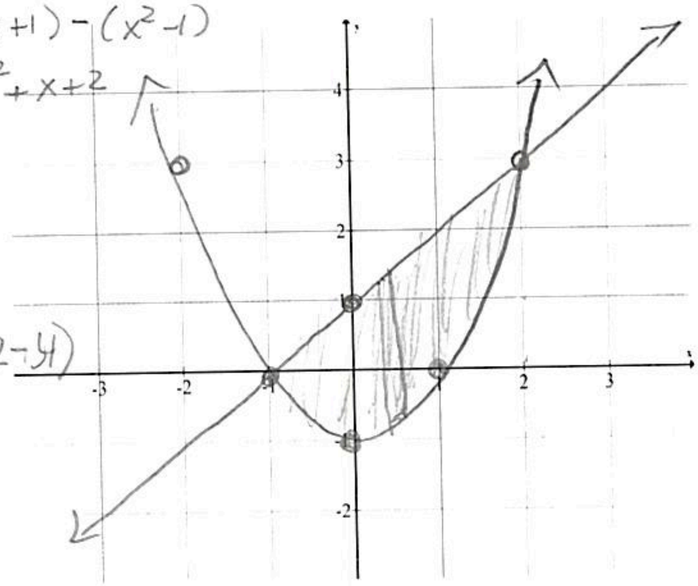
$S = -x^2 + x + 2$

$V = \int_{-1}^{2} (x^4 - 2x^3 - 3x^2 + 4x + 4)\,dx$

$V = \frac{x^5}{5} - \frac{x^4}{2} - x^3 + 2x^2 + 4x \Big]_{-1}^{2}$

$(\frac{32}{5} - 8 - 8 + 8 + 8) - (-\frac{1}{5} - \frac{1}{2} + 1 + 2 - 4)$

$\frac{33}{5} + \frac{1}{2} - 3 + 4 = \boxed{8.1}$



2. Rectangles of height 1 $A = bh$

$V \int_{-1}^{2} (-x^2 + x + 2)\,dx = \boxed{4.5}$

3. Semicircles $A = \frac{\pi r^2}{2}$ $r = \frac{S}{2} = \frac{(-x^2 + x + 2)}{2}$

$V = \frac{\pi}{2} \int_{-1}^{2} \left(\frac{-x^2 + x + 2}{2}\right)^2 dx = \boxed{3.181}$

4. Equilateral triangles $A = \frac{\sqrt{3}}{4} S^2$

$S = (x+1) - (x^2-1)$

$S = -x^2 + x + 2$

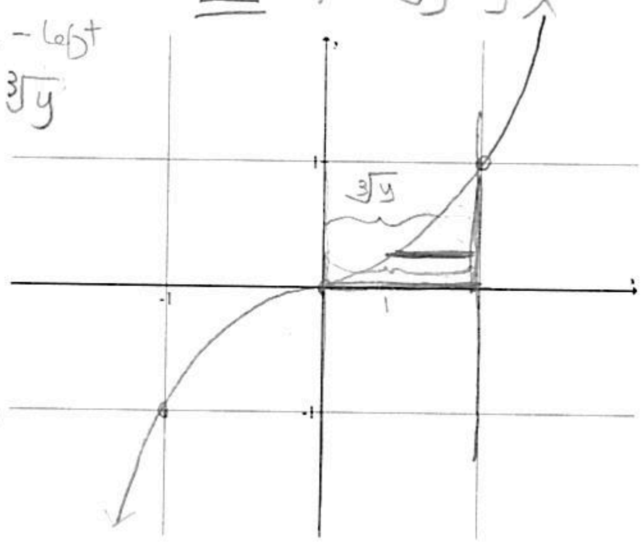
$V = \frac{\sqrt{3}}{4} \int_{-1}^{2} (-x^2 + x + 2)^2\,dx =$

$\boxed{3.507}$

Find the volume of the solid whose base is bounded by the graphs of $y = x^3$ and $y = 0$ and $x = 1$ with the indicated cross sections taken perpendicular to the y-axis. $x = \sqrt[3]{y} = y^{1/3}$

5. Squares $A = s^2$ $s = Rt - Left$

$s = 1 - \sqrt[3]{y}$



$$V = \int_0^1 (1 - \sqrt[3]{y})^2 \, dy = \boxed{.099}$$

or .1 :)

6. Rectangles of height 1

$A = bh$ $b = 1 - \sqrt[3]{y}$

$$V = \int_0^1 (1 - \sqrt[3]{y}) \, dy = \boxed{.249 \text{ or } .25}$$

7. Semicircles

$A = \frac{\pi r^2}{2}$ $r = \frac{(1 - \sqrt[3]{y})}{2}$

$$V = \frac{\pi}{2} \int_0^1 \left(\frac{1 - \sqrt[3]{y}}{2}\right)^2 dy = \boxed{.039}$$

8. Equilateral triangles

$A = \frac{\sqrt{3}}{4} s^2$ $s = 1 - \sqrt[3]{y}$

$$V = \frac{\sqrt{3}}{4} \int_0^1 (1 - \sqrt[3]{y})^2 \, dy = \boxed{.043}$$