CALCULUS BC
WORKSHEET 1 ON LOGISTIC GROWTH

Work the following on **notebook paper**. Do not use your calculator.

1. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

(a) Given $P(0) = 100$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) Find $\frac{d^2P}{dt^2}$ and use it to find the values of P for which the solution curve is concave up and concave down.
Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0) = 1500$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(c) Given $P(0) = 3000$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(d) How many bears are in the park when the population of bears is growing the fastest?

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2. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt} = P\left(3 - \frac{P}{2000}\right)$. What is $\lim_{t \to \infty} P(t)$? What does this number represent in the context of this problem?

TURN->>>

3. (From the 1998 BC Multiple Choice)

The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

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4. Suppose a population of wolves grows according to the logistic differential equation

$\frac{dP}{dt} = 3P - 0.01P^2$,

where P is the number of wolves at time t in years. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest at $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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5. Suppose that a population develops according to the logistic equation

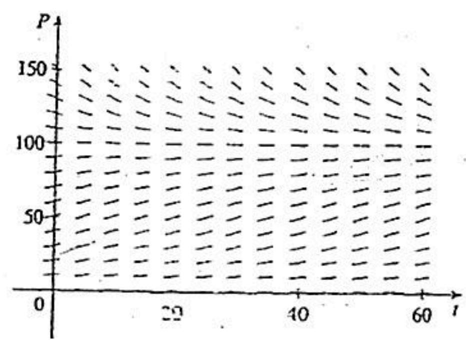
$$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

where t is measured in weeks.

(a) What is the carrying capacity?

(b) A slope field for this equation is shown at the right.
Where are the slopes close to 0?
Where are they largest?
Which solutions are increasing?
Which solutions are decreasing?

(c) Use the slope field to sketch solutions for initial populations of 20, 60, and 120.
What do these solutions have in common?
How do they differ?
Which solutions have inflection points?
At what population level do they occur?



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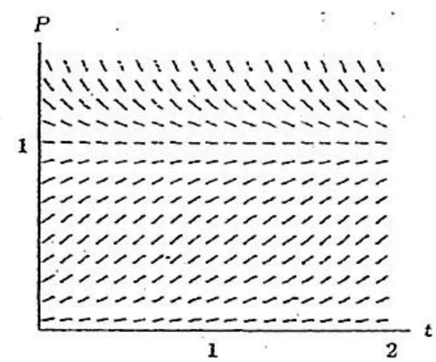
6. (a) On the slope field shown on the right for $\frac{dP}{dt} = 3P - 3P^2$, sketch three solution curves showing different types of behavior for the population P.

(b) Describe the meaning of the shape of the solution curves for the population. Where is P increasing?



Decreasing?
What happens in the long run?
Are there any inflection points?
Where?
What do they mean for the population?

CALCULUS BC
WORKSHEET 2 ON LOGISTIC GROWTH

Work the following on **notebook paper**. Use your calculator on 2(b) and (c), 3(c) and (d), 4(b) and (c), and 5(c) and (d) only.

1. Suppose you are in charge of stocking a fish pond with fish for which the rate of population growth is modeled

by the differential equation $\frac{dP}{dt} = 8P - 0.02P^2$.

(a) Given $P(0) = 50$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) Find $\frac{d^2P}{dt^2}$ and use it to find the values of P for which the solution curve is concave up and concave down. Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0) = 300$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(c) Given $P(0) = 500$.

(i) Find $\lim_{t \to \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

2. A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) and your graphing calculator to find P when $t = 3$ years.

(c) Use your answer to (a) and your graphing calculator to find t when $P = 80$ animals.

TURN->>>

3. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest? Justify your answer.

(b) If $P(0) = 5$, solve for P as a function of t.

(c) Use your answer to (b) and your graphing calculator to determine how many hours have passed when half the student body has heard the rumor.

(d) Use your answer to (b) and your graphing calculator to determine how many students have heard the rumor after 2 hours.

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4. Suppose a rumor is spreading at a dance attended by 200 students. The rumor is spreading at a rate that is directly proportional to both the number of students who have heard the rumor and the number of students who have not heard the rumor. Let P be the number of students who have heard the rumor, and let t be the time in minutes since the rumor began to spread.

(a) Write a differential equation to model this rate of change.

(b) If $P(0) = 10$ and $P(15) = 50$, solve for P as a function of t.

(c) Use your solution to (b) to find the number of students who have heard the rumor after 1 hour.

(d) Use your solution to (b) to find the time it takes for 175 students to hear the rumor.