NAME ________________________________ PERIOD __________

CALCULUS BC

WORKSHEET ON INTERMEDIATE VALUE THEOREM

Work the following on notebook paper.

On problems 1 – 4:

(a) Determine if the Intermediate Value Theorem holds for the given value of k.

(b) If the theorem holds, find a number c for which $f(c) = k$. If the theorem does not hold, give the reason.

(c) Draw a sketch of the curve and the line $y = k$.

1. $f(x) = \frac{1}{x+1}$, $[a, b] = [0, 2]$, $k = \frac{1}{5}$
2. $f(x) = \frac{1}{x+1}$, $[a, b] = [0, 2]$, $k = \frac{3}{4}$
3. $f(x) = x^2 - 3x - 4$, $[a, b] = [-2, 3]$, $k = -2$
4. $f(x) = \sqrt{9 - x^2}$, $[a, b] = [-3, 1]$, $k = 2$

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5. Given the function $f(x) = x^2 + 2x - 5$.

(a) Does $f(x) = 7$ somewhere on the interval $[-1, 3]$? Use the Intermediate Value Theorem to show why or why not.

(b) Does $f(x) = 12$ somewhere on the interval $[-1, 3]$? Use the Intermediate Value Theorem to show why or why not.

6. Use the Intermediate Value Theorem to show that $x^3 + x = 0$ has a root in the interval $[-1, 2]$.

7. One night in January, the outside temperature at midnight was 42°F. At 10 AM the next morning, the temperature was 57°F.

(a) Must there have been a time between midnight and 10 AM when the temperature was 50°F? Explain how you know.

(b) Must there have been a time between midnight and 10 AM when the temperature was 40°F? Explain how you know.

(c) Could there have been a time between midnight and 10 AM when the temperature was 40°F? Explain how you know.

8. One afternoon you were driving on Hwy 290, headed for College Station. At 2 PM, you were driving 60 miles per hour. At 3 PM, you were driving 55 miles per hour.

(a) Must there have been a time between 2 PM and 3PM when you were driving 57 miles per hour? Explain how you know.

(b) Must there have been a time between 2 PM and 3PM when you were driving 45 miles per hour? Explain how you know.

(c) Must there have been a time between 2 PM and 3PM when you were driving 45 miles per hour? Explain how you know.

CALCULUS
WORKSHEET ON CONTINUITY AND INTERMEDIATE VALUE THEOREM

Work the following on **notebook** **paper**.
On problems 1 – 4, sketch the graph of a function f that satisfies the stated conditions.

1. f has a limit at $x = 3$, but it is not continuous at $x = 3$.
2. f is not continuous at $x = 3$, but if its value at $x = 3$ is changed from $f(3) = 1$ to $f(3) = 0$, it becomes continuous at $x = 3$.
3. f has a removable discontinuity at $x = c$ for which $f(c)$ is undefined.
4. f has a removable discontinuity at $x = c$ for which $f(c)$ is defined.

On problems 5 – 7, use the definition of continuity to prove that the function is discontinuous at the given value of a. Sketch the graph of the function.

5. $f(x) = \dfrac{x^2 - 5x + 4}{x - 1}$, $a = 1$

6. $g(x) = \begin{cases} \dfrac{x^2 - 3x}{x^2 - 9} & \text{if } x \neq 3 \\ 1 & \text{if } x = 3 \end{cases}$ $\quad a = 3$

7. $h(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$ $\quad a = 0$

On problems 8 – 9, use the definition of continuity to find the values of k and/or m that will make the function continuous everywhere.

8. $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$

9. $g(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+3) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$

On problems 10 – 12, a function f and a closed interval $[a, b]$ are given. Show whether the conditions of the Intermediate Value Theorem hold for the given value of k. If the conditions hold, find a number c such that $f(c) = k$. If the theorem does not hold, give the reason. Whether the theorem holds or not, sketch the curve and the line $y = k$.

10. $f(x) = 2 + x - x^2$
    $[a, b] = [0, 3]$
    $k = 1$

11. $f(x) = \sqrt{25 - x^2}$
    $[a, b] = [-4.5, 3]$
    $k = 3$

12. $f(x) = \dfrac{1}{x - 2}$
    $[a, b] = [3, 5]$
    $k = \dfrac{5}{6}$

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13. Use the Intermediate Value Theorem to show that $f(x) = x^3 + x$ takes on the value 9 for some x in $[1, 2]$.

On problems 14 – 15, use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

14. $x^4 + x - 3 = 0$ $\quad [1, 2]$

15. $\cos x = x$ $\quad \left[0, \dfrac{\pi}{2}\right]$

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16. Jesse and Kay ran a 1000-m race. One minute after the race began, Jesse was running 20 km/hr, and Kay was running 15 km/hr. Three minutes after the race began, Jesse has slowed to 17km/hr, and Kay had speeded up to 19 km/hr. Assume that each runner's speed is a continuous function of time. Prove that there is a time between 1 minute and 3 minutes after the race began at which each one was running exactly the same speed. Is it possible to tell what that speed is? Is it possible to tell when that speed occurred? Explain.