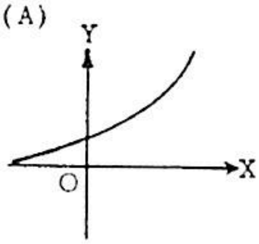
slope+ concave down

D3 9/36

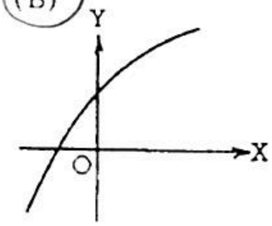
1. '69 AB #16

If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

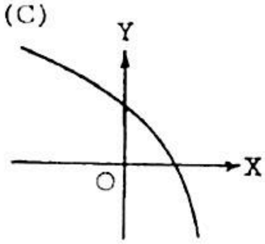
(A)



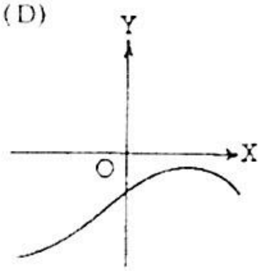
(B)



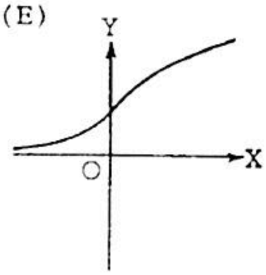
(C)



(D)



(E)

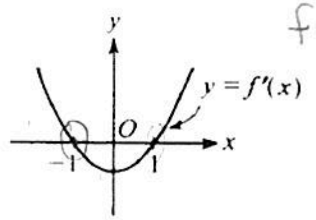


2. '85 AB #31

The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

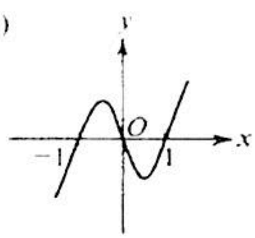
3. '85 AB #33



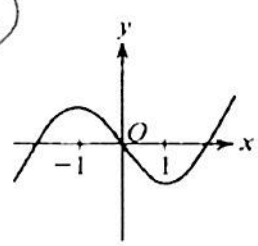
f: +slope $(-\infty,-1) \cup (1,\infty)$
- slope $(-1,1)$
slope $= 0$ @ -1 + 1

The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

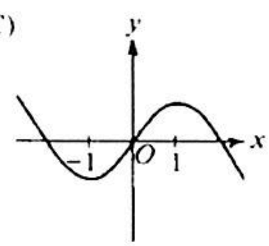
(A)



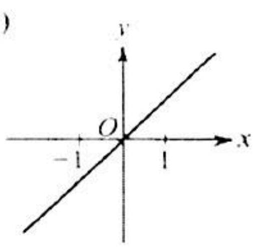
(B)



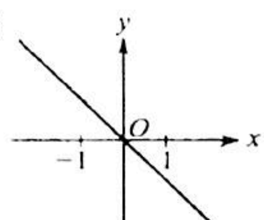
(C)



(D)



(E)



$y = -10(x-2)$ $y'' = 0$ @ $x = 2$ $y' = \frac{-(-5)}{(x-2)^2} = \frac{5}{(x-2)^2}$

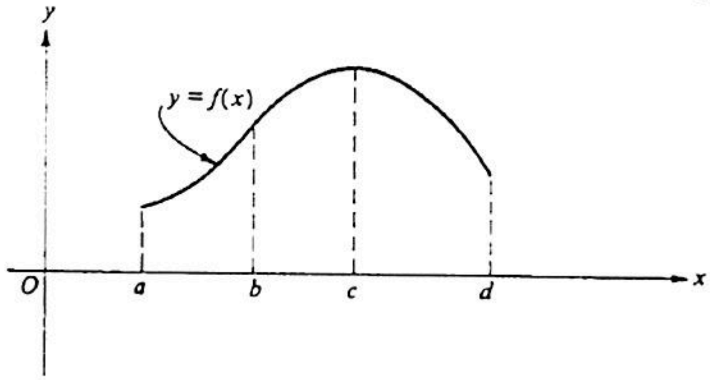
$+ \quad 2 \quad -$

4. '88 AB #4

The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that $y'' = \frac{-5(2)(x-2)}{(x-2)^4}$

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

5. '88 AB #8



The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

+ slope concave down

I. $a < x < b$

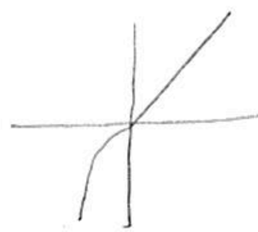
II. $b < x < c$

III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

6. '93 AB #19

Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0. \\ x & \text{for } x > 0. \end{cases}$ $f'(x) = 3x^2$ $f'(0) = 0$; $f'(x) = 1$ $f'(0) = 1$ Which of the following statements about f is true?



(A) f is an odd function.

(B) f is discontinuous at $x = 0$.

(C) f has a relative maximum.

(D) $f'(0) = 0$

(E) $f'(x) > 0$ for $x \neq 0$

7. '97 AB #5

The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48$

$12(3x^2 - 8x + 4) = 0$

$12(3x - 2)(x - 2) = 0$

$x = 2/3$ $x = 2$

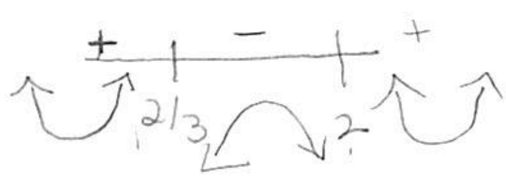
(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

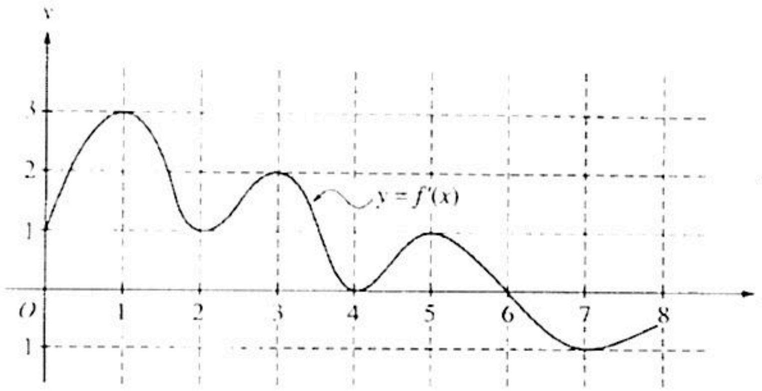
(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$



#8 & 9 refer to the graph and the information below.

'97 BC
#7 & 8



$f(3) = 5$
$f'(3) = 2$

The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown above.

8. The point $(3, 5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3, 5)$ is

$y - 5 = 2(x - 3)$

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y - 5 = 2(x + 3)$

(E) $y + 5 = 2(x + 3)$

9. How many points of infelction does the graph of f have? $f''(x)$ is slope of $f'(x)$ above

→ $f''(x)$ chgs signs

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

10. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

'93 AB
#15

$f'(x) = (x-3)^2 + 2(x-3)(x-2) = 3x^2 - 16x + 21$ $(3x-7)(x-3)$

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

\+ − +
7/3 3

11. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

'98 BC
#1

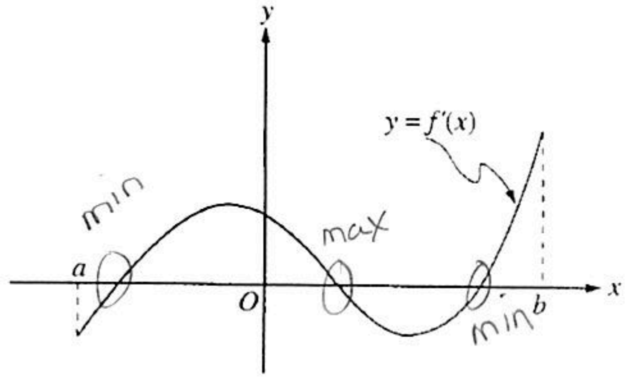
$f'(x) = 3x^2 + 6x - 9$
$3(x^2 + 2x - 3)$
$(x+3)(x-1) = 0$

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

\+ − +
−3 1

12.

'97 BC
#12



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

13.

'98 BC
#16

If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f? $f'(x) = 15x^4 - 20x^3$ $f''(x) = 60x^3 - 60x^2$
$= 60x^2(x-1)$

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1

14.

'03 BC
#83

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

The function *f* is continuous and differentiable on the closed interval [0, 4]. The table above gives selected values of *f* on this interval. Which of the following statements must be true?

(A) The minimum value of *f* on [0, 4] is 2.

(B) The maximum value of *f* on [0, 4] is 4.

(C) $f(x) > 0$ for 0 < x < 4.

(D) $f'(x) < 0$ for 2 < x < 4.

(E) There exists *c*, with 0 < *c* < 4, for which $f'(c) = 0$. → $f(0) = 2$ $f(4) = 2$ Rolle's Theorem

15. The height *h*, in meters, of an object at time *t* is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

'03 BC
#91

(A) 2.545 meters

(B) 10.263 meters

(C) 34.125 meters

(D) 54.889 meters

(E) 89.005 meters

use calc

$v(t) = 24 + 36\sqrt{t} - 32t = 0$

$6 + 9\sqrt{t} - 8t = 0$

@ $t = 2.5445657$

$h(2.5445) = 54{,}889$

↓

$v(t)$ Δ from + to −

Max

2005 AB #4

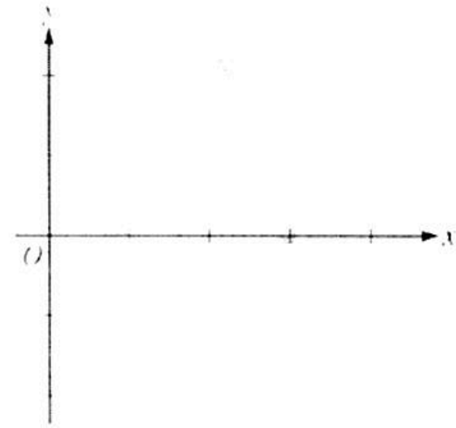
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)

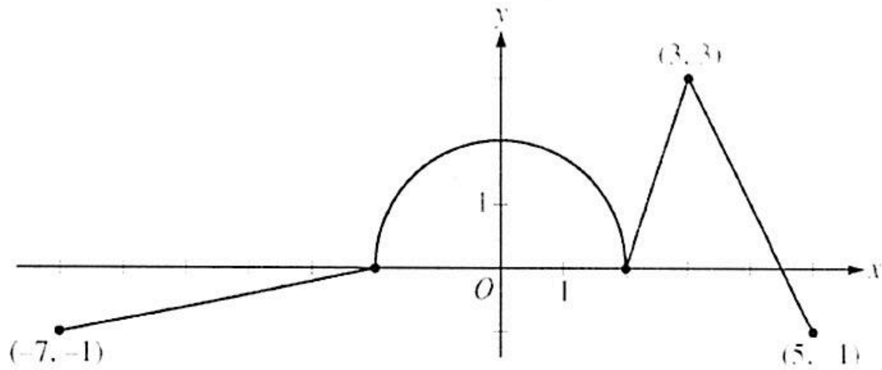


(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

a) $f(x)$ has a relative max @ $x=2$ b/c $f'(x)$ changes + to −

$f(x)$ could be 



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

$g'(3) = 3$
$g'(0) = 2$
$(0, 5)$ $(3, y)$
$m = 3$
$\frac{y-5}{3-0} = 3$

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

b) $g(x)$ has points of inflection at $x = 0$, $x = 2$ + $x = 3$ because $g'(x)$ changes from increasing to decreasing ($g''(x)$ changes from + to −) @ $x = 0$ + $x = 3$ + $g'(x)$ changes from decreasis to increasing ($g''(x)$ chgs from − to +) @ $x = 2$