7.2 Volume: Disk and Washer Methods – day 1 (rotations around an axis)

If a region in the plane is revolved about a line, the resulting solid is a <u>solid of revolution</u>, and the line is called the <u>axis of revolution</u>.

Volume of disk = (area of disk)(width of disk) =

The Disk Method

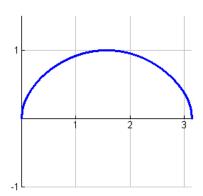
Horizontal Axis of Revolution

$$V = \pi \int_{a}^{b} [R(x)]^{2} dx$$

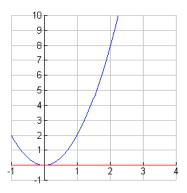
Vertical Axis of Revolution

$$V = \pi \int_{c}^{d} [R(y)]^{2} dy$$

1) Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis $(0 \le x \le \pi)$ about the x-axis.

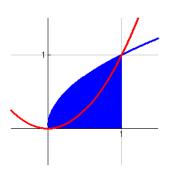


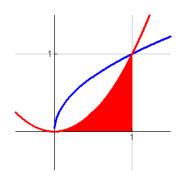
2) Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2$, y = 0 and x = 2 about the x-axis.

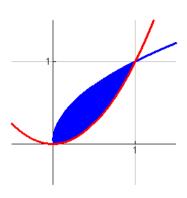


3) Find the volume of the solid generated by revolving the region bounded by the graphs $x = y^2$, x = 0, y = 0 and y = 1 about the y-axis.

The Washer Method







The Washer Method Horizontal Axis of Revolution

$$V = \pi \int_{a}^{b} [R(x)]^{2} dx - \pi \int_{a}^{b} [r(x)]^{2} dx$$

R(x) = Top - Bottom of Outer Region

r(x) = Top - Bottom of Inner Region

Vertical Axis of Revolution

$$V = \pi \int_{c}^{d} [R(y)]^{2} dy - \pi \int_{c}^{d} [r(y)]^{2} dy$$

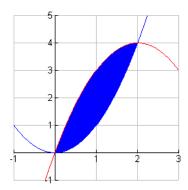
R(y) = Right - Left of Outer Region

r(y) = Right - Left of Inner Region

1) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis.



2) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2$ and $y = 4x - x^2$ about the x-axis.



3) Find the volume of the solid formed by revolving the region bounded by the graphs of $x = 3 - y^2$ and x = 1 about the y-axis.

