Chapter 5: Differential Equations

WORKSHEET 1 ON DIFFERENTIAL EQUATIONS

An equation involving a derivatives is called a **differential equation**. A differential equation in the form $\frac{dy}{dx} = f(y)g(x)$ is called **separable.** To solve separable differential equations:

1) Separate the variables.
2) Integrate both sides.
3) If there is an initial condition, substitute in and solve for C.
4) Use the C you found and solve for f(x).

Ex1: **FR Find a solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ satisfying $f(0) = \frac{1}{2}$.

Ex 2: ** If $\frac{dy}{dx} = 2y^2$ and y=-1 when x=1, then when x=2, y=?

Ex 3: Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with initial condition $f(3) = \frac{1}{4}$

Work the following on **<u>notebook paper</u>**. Do not use your calculator.

Solve for y as a function of x.

1. $\frac{dy}{dx} = \frac{x-3}{y}$ and $y(2) = -5$

2. $y' = 2x\sqrt{y}$ and $y(2) = 25$

3. $\frac{dy}{dx} = 4y^2 \sec^2(2x)$ and $y\left(\frac{\pi}{8}\right) = 1$

4. $xy\frac{dy}{dx} = \ln x$ and $y(1) = 2$

5. $y' = 2x \sec y$ and $y(2) = -\frac{\pi}{2}$

6. $y' - xe^y = 2e^y$ and $y(0) = 0$

7. $\frac{dy}{dx} = 2xy^3 \sin(x^2)$ and $y(0) = -1$

8. $\frac{dy}{dx} = \frac{1}{y^2}$ and $y(0) = 4$

9. Find a curve in the xy-plane that passes through the point (0, 3) and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$.

CALCULUS BC
WORKSHEET 2 ON DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Do not use your calculator.

Solve for y as a function of x.

1. $\frac{dy}{dx} = 6x^2 y$ and $y(0) = 4$

2. $\frac{dy}{dx} = \frac{1+x}{\sqrt{y}}$ and $y(2) = 9$

3. $\frac{dy}{dx} = (y-2)^2 \cos(3x)$ and $y\left(\frac{\pi}{2}\right) = 3$

4. $\frac{dy}{dx} = 3x + xy$ and $y(4) = -2$

5. $\frac{dy}{dx} = \frac{y-3}{x^2}$, $x \neq 0$, and $y(4) = 0$

6. $\frac{dy}{dx} = x^2 y + 2x^2$, $y(-1) = 4$

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7. If $\frac{dy}{dx} = y\cos x$ and $y = 3$ when $x = 0$, then $y = ?$

8. Find an equation of the curve that satisfies $\frac{dy}{dx} = 4x^3 y$ and whose y-intercept is 7.

Multiple choice. Solve. All steps must be shown.

9. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

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10. Consider the differential equation $\frac{dy}{dx} = xy^3$. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = -1$, and state its domain.

11. Consider the differential equation $\frac{dy}{dx} = \frac{y+5}{x}$, $x \neq 0$. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-3) = 1$, and state its domain.

CALCULUS
WORKSHEET ON APPLICATIONS OF DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Use your calculator and give decimal answers correct to three decimal places.

1. A pie is removed from an oven at 450° and left to cool at a room temperature of 70°. After 30 minutes, the pie's temperature is 200°. How many minutes after being removed from the oven will the temperature of the pie be 100°?

2. A certain population increases at a rate proportional to the square root of the population. If the population goes from 2500 to 3600 in five years, what is the population at the end of t years?

3. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts at 36 in. and drops to 35 in. in one hour, how long will it take for all of the water to leak out of the barrel?

4. A student studying a foreign language has 50 verbs to memorize. The rate at which the student can memorize these verbs is proportional to the number of verbs remaining to be memorized, $50 - y$, where the student has memorized y verbs. Assume that initially no verbs have been memorized and suppose that 20 verbs are memorized in the first 30 minutes.
   (a) How many verbs will the student memorize in two hours?
   (b) After how many hours will the student have only one verb left to memorize?

5. Let $P(t)$ represent the number of wolves in a population at time t years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.
   (a) If $P(0) = 500$, find $P(t)$ in terms of t and k.
   (b) If $P(2) = 700$, find k.
   (c) Find $\lim_{t \to \infty} P(t)$.

Solve. Do not use your calculator.

6. $y' - xy^2 = 0$ and $y(1) = 1$

7. $\frac{dy}{dx} - 2xy = 2x$ and $y(0) = 3$

8. $\frac{dy}{dx} = \frac{3x}{y}$ and $y(1) = -3$

9. $2x\frac{dy}{dx} - \ln\left(x^2\right) = 0$ and $y(1) = 3$