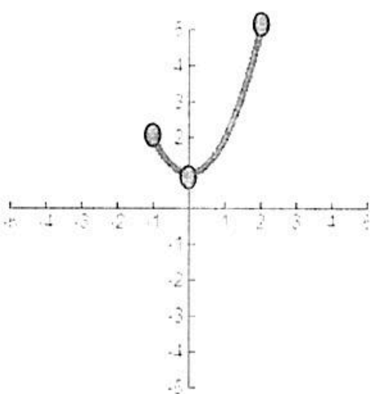
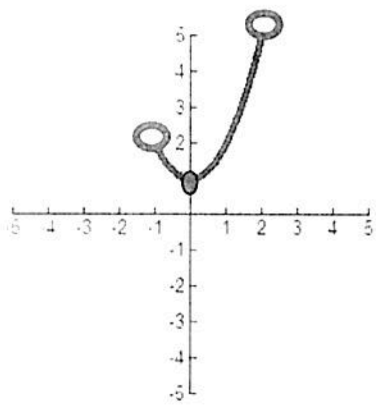
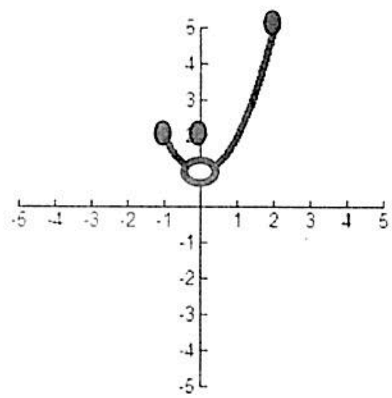
AP Calculus Name ____________________

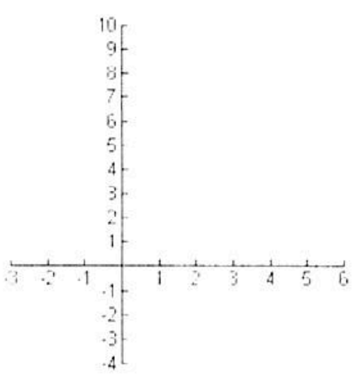
4.1 Extrema on an Interval

The minimum and maximum of a function on an interval are the extreme values, or extrema of the function on the interval.

The Extreme Value Theorem
If f is continuous on a closed interval [a, b], then f has both a minimum and a maximum on the interval.

1) Locate the absolute extrema of the function $f(x) = x^2 - 2x$ over each interval.



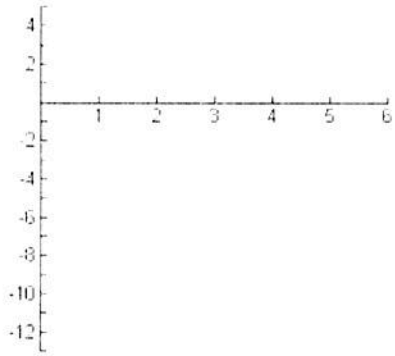
a) [-1, 2]

b) (1, 3]

c) (0, 2)

d) [1, 4)

2) Sketch the graph of $f(x) = \begin{cases} 2 - x^2, & 1 \le x < 3 \\ 2 - 3x, & 3 \le x \le 5 \end{cases}$. Then locate the absolute extrema of the function over the interval [1, 5].



Relative Extrema and Critical Numbers

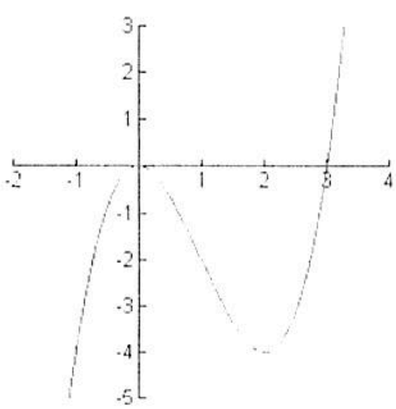
Relative maximum: occurs on a "hill"

Relative minimum: occurs in a "valley"

Hills (relative maximum) and valleys (relative minimum) can occur in two ways:

1) If the hill or valley is smooth and rounded, the graph has a _______________ tangent line at the high point or low point. (slope = ___________)

2) If the hill or valley is sharp and peaked, the graph represents a function that is _______ ___________________ at the high or low point. (slope = _____________)



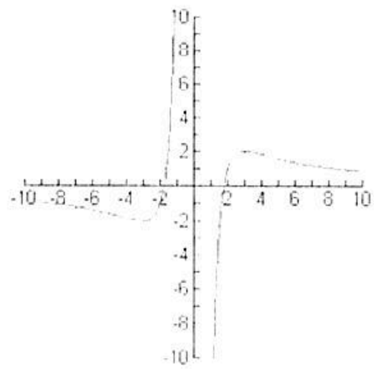
$f(x) = x^3 - 3x^2$

relative minimum: ________________

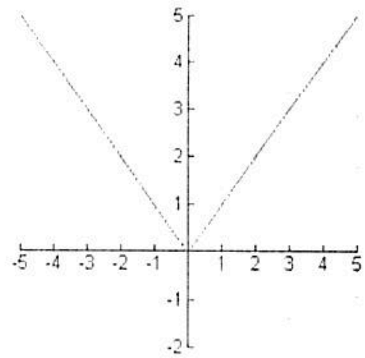
relative maximum: ________________

Find the value of the derivative at each of the relative extrema given.

3) $f(x) = \dfrac{9(x^2 - 3)}{x^3}$ (3,2) and (-3, -2)



4) $f(x) = |x|$ (0, 0)



At relative extrema the derivative is either ____________ or ________________. The x-values at these special points are called critical numbers.

Definition of a critical number

Critical numbers are the x-values of a function that make the first derivative equal 0 or undefined.

These are the x-values of where the graph of function "turns": if the derivative equals 0 the graph will have a __________________turn, if the derivative is undefined the graph will have a __________________ turn.

Finding Extrema on a Closed Interval

1) Find the critical numbers of f in (a, b)
2) Evaluate f at each critical number in (a, b)
3) Evaluate f at each endpoint of [a, b]
4) The least of these values is the minimum. The greatest is the maximum.

5) Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval [-1, 2].

6) Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval [-1, 3].

7) Find the extrema of $f(x) = 4 \sin x - 3$ on the interval $[0, 2\pi]$.

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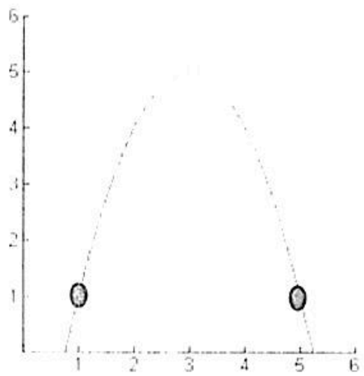
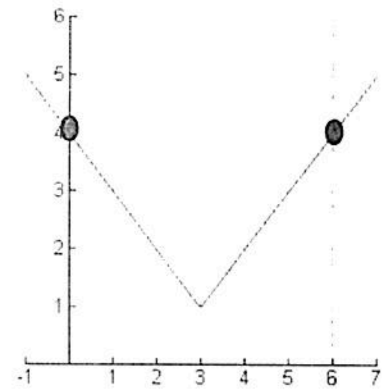
4.2 Rolle's Theorem and the Mean Value Theorem

Rolle's Theorem

Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a) = f(b) then there is at least one number c in (a, b) such that f'(c) = 0.





Determine whether Rolle's Theorem can be applied to f(x) on the given interval. If it can, find all values of c in the open interval (a, b) where f'(c) = 0.

1) $f(x) = x^2 - 3x + 2$ [1, 2]

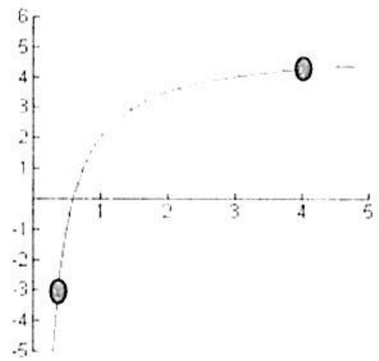
2) $f(x) = x^4 - 2x^2$ [-2, 2]

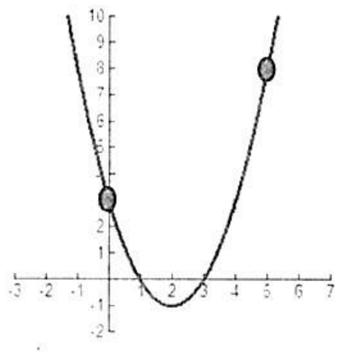
3) $f(x) = (x - 3)(x + 1)^2$ [-1, 3]

The Mean Value Theorem

If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$





Determine whether the Mean Value Theorem can be applied to f(x) on the closed interval [a, b]. If the MVT does apply, find all values of c in the open interval (a, b) for which it applies.

4) $f(x) = x(x^2 - x - 2)$ [-1, 1]

5) $f(x) = -x^2 - x + 6$ [-2, 2]

6) $f(x) = x^3 - 7x + 6$ [1, 3]

Homework – worksheet (practice)

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How AP will test Theorems (3.2)

1) The function f is continuous for $-4 \le x \le 2$ and differentiable for $-4 < x < 2$ and let $f(-4) = -6$ and $f(2) = 6$.

For each choice below, see if you can identify which theorem is being represented (IVT, EVT, MVT, Rolle's). Then try to decide if the statement is true or false.

(A) There exists c, where $-4 < c < 2$, such that $f(c) = 3$

(B) There exists c, where $-4 < c < 2$, such that $f(c) = 0$.

(C) There exists c, where $-4 < c < 2$, such that $f'(c) = 0$.

(D) There exists c, where $-4 < c < 2$, such that $f'(c) = 2$.

(E) There exists c, where $-4 < c < 2$, such that $f(c) \ge f(x)$ for all x on the closed interval $-4 \le x \le 2$.

(F) There exists c, where $-4 < c < 2$, such that $f(c) \le f(x)$ for all x on the closed interval $-4 \le x \le 2$.

2) Let f be a function that is differentiable on the open interval (1, 10) and let $f(3) = -6$, $f(7) = 6$, and $f(9) = -6$.

For each choice below, see if you can identify which theorem is being represented (IVT, EVT, MVT, Rolle's). Then try to decide if the statement is true or false.

I. f has at least 2 zeros.

II. The graph of f has at least one horizontal tangent.

III. For some *c*, $3 < c < 7$, $f(c) = 4$.

IV. For some *c*, $3 < c < 7$, $f'(c) = 4$.

V. For some *c*, $7 < c < 9$, $f(c) = -8$.

VI. For some *c*, $7 < c < 9$, $f'(c) = -6$.