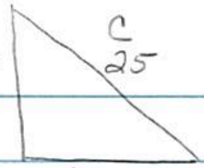
Related Rates W/S

① 

Find: db/dt

When: $a=7$

Given: $da/dt = -3$ ft/min

Equ: $a^2 + b^2 = c^2$

$2a\,da/dt + 2b\,db/dt = 2c\,dc/dt$

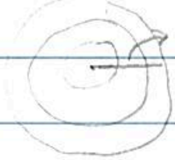
$2(7)(-3) + 2(24)\,db/dt = 2(25)(0)$ ($dc/dt = 0 \rightarrow$ ladder Length doesn't chg)

$-42 + 48\,db/dt = 0$

$48\,db/dt = 42$

$\boxed{db/dt = 7/8 \text{ ft/min}}$

when top of ladder is 7 ft from ground, the distance from wall & bottom of ladder is chg'g at rate of 7/8 ft/min

② 

Find: dA/dt

When: $r = 4$ ft

Given: $dr/dt = 1$ ft/sec

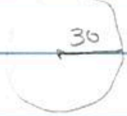
$A = \pi r^2$

$dA/dt = 2\pi r\,dr/dt$

$dA/dt = 2\pi(4)(1)$

$\boxed{dA/dt = 8\pi \text{ ft}^2\text{/sec}}$

when radius is 4 ft, area of circle is chang's at rate of 8π ft²/sec.

③ 

Find: dr/dt

When: $r = 30$ cm

Given: $dv/dt = 800$ cm³/min

Eq: $V = 4/3\,\pi r^3$

$V = \frac{4}{3}\pi r^3$

$dv/dt = 4\pi r^2\,dr/dt$

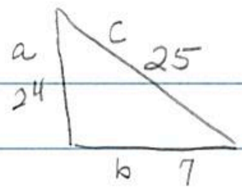
$800 = 4\pi(30)^2\,dr/dt$

$dr/dt = \frac{800}{3600\pi}$

$\boxed{dr/dt = \frac{2}{9\pi} \text{ cm/min}}$

When radius is 30 cm, radius is chg'g at a rate of $2/9\pi$ cm/min.

(4)



Find da/dt

When $b=7$

Given $db/dt = 2$ ft/sec

Equ.: $a^2+b^2=c^2$

$(dc/dt = 0)$

$a^2+b^2=c^2$

$2a\,da/dt + 2b\,db/dt = 2c\,dc/dt$

$2(24)\,da/dt + 2(7)(2) = 0$

$48\,da/dt + 28 = 0$

$48\,da/dt = -28$

$\boxed{da/dt = \boxed{-7/12 \text{ ft/sec}}}$

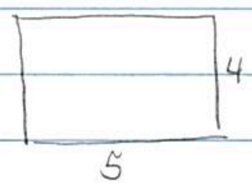
when base is 7 ft from wall, the ladder is <u>falling</u> at a rate of 7/12 ft/sec.

(5) $2x - 2y\,dy/dt = 0$

$dy/dt = \frac{x}{y} = \frac{5}{3}$ $\quad M_N = -3/5$

$\boxed{y-3 = -\frac{3}{5}(x-5)}$

(6)



Find: dA/dt

When: $w=4$ & $L=5$

Given: $dw/dt = 2$ cm/sec

$dL/dt = 3$ cm/sec.

Equation: $A = LW$

$A = LW$

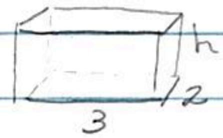
$dA/dt = W\,dL/dt + L\,dw/dt$

$= 4(3) + 5(2)$

$dA/dt = 12 + 10 = \boxed{22 \text{ cm}^2/\text{sec}}$

when the width is 4 & length is 5, the area is changing at a rate of 22 cm²/sec.

(7)



Find dh/dt

When : — $(B = 6)$

Given: $dv/dt = 3\ ft^3/min$

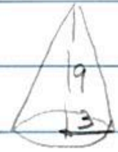
Equ: $V = Bh \rightarrow B$ = area of base → never chgs.

$dv/dt = dB/dt\, h + B\, dh/dt$

$3 = 0 + 6\, dh/dt$

$$\boxed{dh/dt = 1/2\ ft/min}$$

(8)



Find dh/dt

When $h = 3$ cm

Given: $dv/dt = -1\ cm^3/min$ (leaking)

Equ: $V = \frac{1}{3}\pi r^2 h$

☆ $\frac{r}{h} = \frac{3}{9}$

$r = \frac{3h}{9} = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi h^3}{9}$

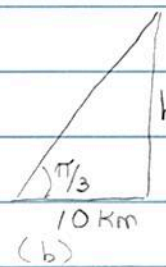
$dv/dt = \frac{\pi h^2}{3}\, dh/dt$

$-1 = 3\pi\, dh/dt$

$$\boxed{dh/dt = -\frac{1}{3\pi}\ cm/min}$$

When the height is 3 cm, the waterlevel is decreas'g at a rate of $\frac{1}{3\pi}$ cm/min.

(9)



$\tan\frac{\pi}{3} = \frac{h}{10}$

$\sqrt{3} = \frac{h}{10}$

$h = 10\sqrt{3}$

Find dh/dt

When $\theta = \pi/3 \rightarrow h = 10\sqrt{3}$

Given $d\theta/dt = 1\ rad/2\ mins$

Equ: $\tan(\theta) = \frac{h}{10}$ $(db/dt = 0)$

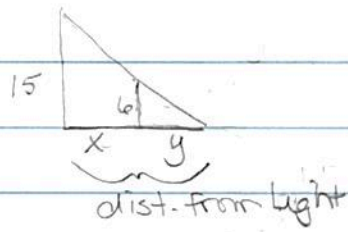
$\tan(\theta) = \frac{1}{10}h$

$\sec^2(\theta)\, d\theta/dt = \frac{1}{10}\, dh/dt$

$\sec^2(\pi/3)(.5) = \frac{1}{10}\, dh/dt$

$dh/dt = 20$ rad/min

10.

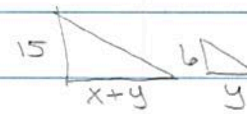


Find $\frac{dx}{dt} + \frac{dy}{dt}$

When: $x = 10$

Given: $dx/dt = 5$ ft/sec.

Equ. $\frac{15}{x+y} = \frac{6}{y}$ → similar Δ's



$\frac{15}{x+y} = \frac{6}{y}$

$15y = 6x + 6y$

$9y = 6x$

$9\,dy/dt = 6\,dx/dt$

$9\,dy/dt = 6(5)$

$dy/dt = 30/9 = 10/3$ ft/sec

$dx/dt + dy/dt = 25/3$ ft/sec

When he is 10 ft from base of light, the tip of his shadow is moving at a rate of 10/3 ft/sec

11.

Find dr/dt

When $r = 3$

Given $dV/dt = 9\pi$ in^3/min

Equ: $V = \frac{1}{3}\pi r^2 h$

$h = 3r$

$V = \frac{1}{3}\pi r^2(3r)$

$V = \frac{1}{3}\pi(3r^3)$

$V = \pi r^3$

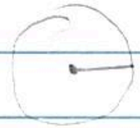
$dV/dt = 3\pi r^2\,dr/dt$

$9\pi = 3\pi(9)\,dr/dt$

$dr/dt = \frac{9\pi}{27\pi} = \frac{1}{3}$ in/min

When radius is 3 in., the radius of the cone is increasing at a rate of 1/3 in/min.

(12)



Find dr/dt

When $V = 36\pi\ m^3$

Given $dV/dt = 3\ m^3/min$

Equ: $V = \frac{4}{3}\pi r^3$

Must find r first!

$36\pi = \frac{4}{3}\pi r^3$

$\frac{108}{4} = r^3 \quad r^3 = 27 \quad r = 3$

$V = \frac{4}{3}\pi r^3$

$dV/dt = 4\pi r^2\, dr/dt$

$3 = 4\pi(9)\, dr/dt$

$dr/dt = \frac{3}{36\pi} = \frac{1}{12\pi}$ m/min

When the volume is $36\pi\ m^3$, the radius is changing $\frac{1}{12\pi}$ m/min.

(13)

Find dh/dt

When $h = 15$ ft

Given $dV/dt = 10\ ft^3/min$

Equ: $V = \frac{1}{3}\pi r^2 h$

$\frac{d}{2} = \frac{3h}{2}$

$r = 3h/2$

$V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h$

$V = 3\pi/4\ h^3$

$dV/dt = \frac{9\pi}{4} h^2\, dh/dt$

$10 = \frac{9\pi}{4}(15)^2\, dh/dt$

$10 = \frac{9\pi}{4}(225)\, dh/dt$

$dh/dt = \frac{40}{2025\pi} = 8/405\pi$ ft/min

When the height is 15 ft, the height is changing at a rate of $8/405\pi$ ft/min.

(14)

Find dv/dt

When? $r=1$

Given $dr/dt = 2$ cm/sec.

Equ. $V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

$4\pi = 4\pi r^2 \quad \boxed{r=1}$

$V = \frac{4}{3}\pi r^3$

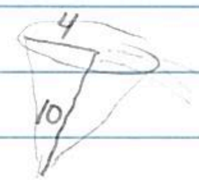
$dv/dt = 4\pi r^2 \, dr/dt$

$dv/dt = 4\pi(1)(2)$

$dv/dt = 8\pi \text{ cm}^3/\text{sec}$

When surface area is 4π cm^2, the volume is changing at a rate of 8π cm^3/sec.

(15)



Find dh/dt

When $h=5$

Given $dv/dt = -10$ ft^3/min

Eq.: $V = \frac{1}{3}\pi r^2 h$

$\frac{4}{10} = \frac{r}{h} \quad 10r = 4h$

$r = \left(\frac{2}{5}h\right)$

$V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$V = \frac{4\pi}{75}h^3$

$dv/dt = \frac{12\pi}{75}h^2 \, dh/dt$

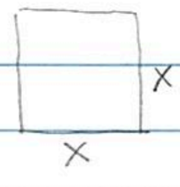
$-10 = \frac{\cancel{12}^{4}\pi}{\cancel{75}_{3}}(25)\, dh/dt$

$-10 = 4\pi \, dh/dt$

$dh/dt = -5\pi/2$ ft/min

When height of water is 5 ft, the water level is changing at a rate of $-\frac{5\pi}{2}$ ft/min.

(16)

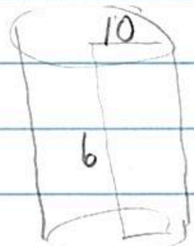


Find dA/dt
When $x=3$
Given $dx/dt = 2$ ft/min
Equ. $A = x^2$

$$dA/dt = 2x\,dx/dt$$
$$= 2(3)(2)$$
$$dA/dt = 12 \text{ ft}^2/\text{min}$$

When the side length is 3ft, the area of the square is increasing by 12 ft²/min.

(17)



Find dV/dt
When: $h=6$ & $r=10$
Given $dh/dt = 1$ in/sec $dr/dt = -1$ in/sec
Equ: $V = \pi r^2 h$

★ product rule!

$$V = \pi r^2 h$$
$$dV/dt = 2\pi r\,dr/dt\,h + \pi r^2\,dh/dt$$
$$= 2\pi(10)(-1)(6) + \pi(10)^2(1)$$
$$= -120\pi + 100\pi$$
$$dV/dt = -20\pi \text{ in}^3/\text{sec}$$

When radius is 10 & height is 6, volume is decreasing at a rate of 20π in³/sec.

or volume is changing at a rate of -20π in³/sec. Since rate of chg is negative, volume is decreasing

(18)

Find dA/dt

When $r=30$

Given $dr/dt = 3$ ft/sec

Equ. $A = \pi r^2$

($r=30$ ft at 10 sec.)

$A = \pi r^2$

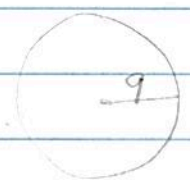
$da/dt = 2\pi r \, dr/dt$

$= 2\pi(30)(3)$

$da/dt = 180\pi$ ft^2/sec

After 10 seconds, the area is changing at a rate of 180π ft^2/sec.

(19)



Find: dv/dt

When $r = 9$ cm

Given $dr/dt = -15$ cm/min

Equ. $V = \frac{4}{3}\pi r^3$

$dv/dt = 4\pi r^2 dr/dt$

$= 4\pi(81)(-15)$

$dv/dt = -4860\pi$ cm^3/min

When radius is 9 cm, volume is decreasing at a rate of 4860π cm^3/min