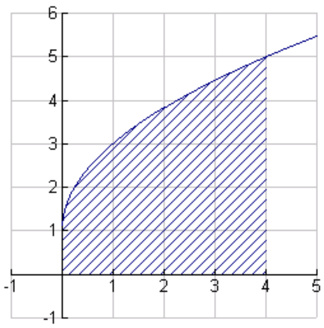
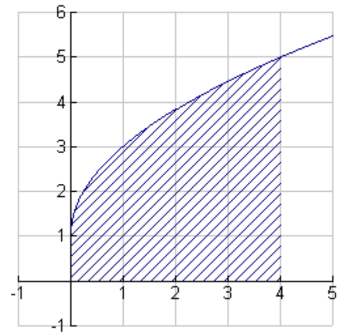
AP Calc BC Worksheet 5.2 and 5.3

1) Bound the area of the shaded region by approximating the left and right sums. Use rectangles of width 1.

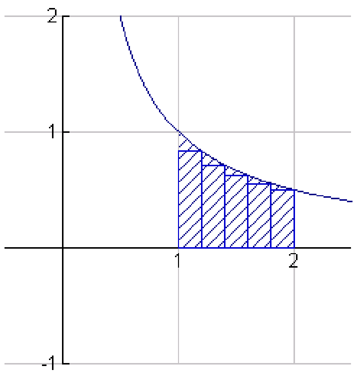
Left Sum

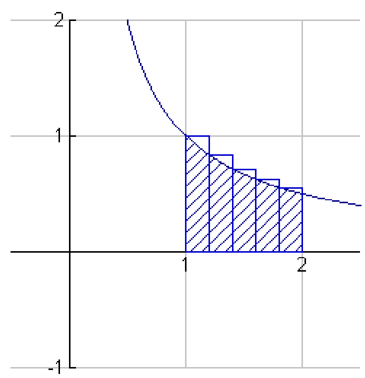


Right Sum



2) Use the upper and lower sums to approximate the area of the region using the given number of subintervals of equal width. $y = \frac{1}{x}$

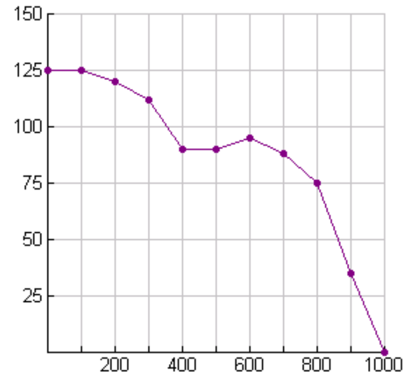




3) Find the right Reimann sum for the area of the region bounded by the graph of $y = \sqrt{x} + 1$ and the x-axis between x = 0 and x = 2, using 8 equal width rectangles.

4) Find the midpoint sum to approximate the area of the region bounded by $f(x) = x^2 + 4x$ over the interval [0, 4] using 4 equal width rectangles.

5. Use the graph and table for the following questions:



L1	L2
0	125
100	125
200	120
300	112
400	90
500	90
600	95
700	88
800	75
900	35
1000	0

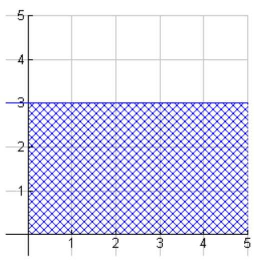
a) Estimate the area using right Riemann sums with 5 equal width rectangles.

b) Estimate the area using left Riemann sums with 5 equal width rectangles.

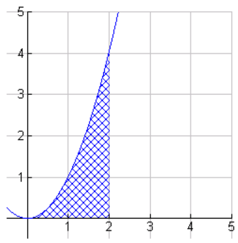
c) Estimate the area using midpoint Riemann sums with 5 subintervals of equal length.

Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

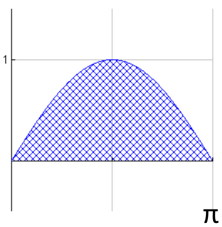
6. $f(x) = 3$



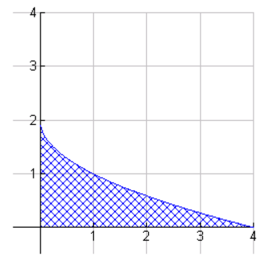
7. $f(x) = x^2$



8. $f(x) = \sin x$



9. $f(y) = (y - 2)^2$



Sketch the region whose area is given by the definite integral. Then use the geometric formula to evaluate the integral (a > 0, r > 0)

10. $\int_0^4 x\,dx$

11. $\int_0^8 (8-x)\,dx$

12. $\int_{-3}^3 \sqrt{9-x^2}\,dx$

For problems #13 – 15, evaluate the integral using the following values.

$\int_2^4 x^3\,dx = 60$	$\int_2^4 x\,dx = 6$	$\int_2^4 dx = 2$

13. $\int_2^2 x^3\,dx$

14. $\int_2^4 (x-8)\,dx$

15. $\int_2^4 \left(6+2x-x^3\right)dx$

16. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$

a. $\int_2^6 \left[g(x)-f(x)\right]dx$

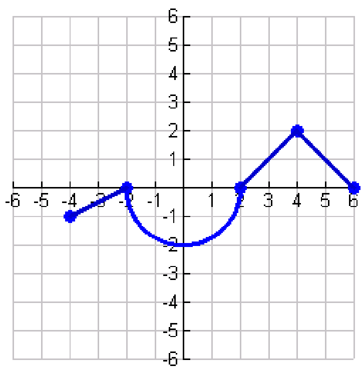
b. $\int_2^6 2g(x)\,dx$

c. $\int_2^6 3f(x)\,dx$

17. Use the table of values to estimate $\int_0^6 f(x)\,dx$. Use three subintervals and the (a) left endpoints, (b) right endpoints, and (c) midpoints. If f is an increasing function, how does each estimate compare with the actual value?

x	0	1	2	3	4	5	6
f(x)	-6	0	8	18	30	50	80

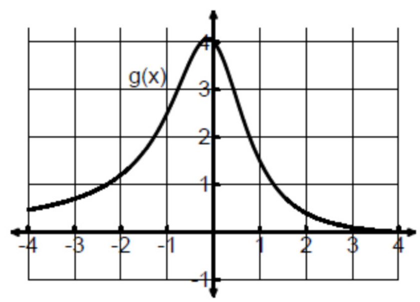
18. The graph of f consists of line segments and a semicircle. Evaluate each definite integral by using geometric formulas.



a) $\int_0^2 f(x)\,dx$ b) $\int_2^6 f(x)\,dx$ c) $\int_{-4}^2 f(x)\,dx$

d) $\int_{-4}^6 f(x)\,dx$ e) $\int_{-4}^6 |f(x)|\,dx$ f) $\int_{-4}^6 [f(x)+2]\,dx$

From NC #48, 49, 50, 51



1. Set up an estimate of the area between the curve g(x) and the x-axis using the right Riemann sums with 4 subintervals of equal width for [-2, 2].

2. Suppose a car has the velocities, in feet per second, represented in the table below. Find the estimate of the total distance for [0, 12] seconds using the midpoints of 3 rectangles.

t (sec)	0	2	4	6	8	10	12
v(t) ft/sec	20	30	38	44	43	35	39

**calc. FR 3. In the table below, v(t) represents the velocity, in ft/sec, of a car traveling on a straight road.

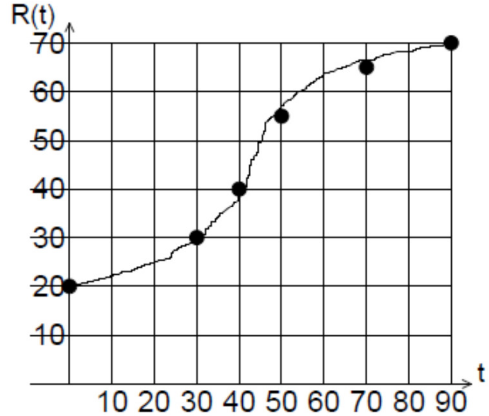
t (sec)	0	5	10	15	20	25	30	35	40	45	50
v(t) ft/sec	0	12	20	30	55	70	78	81	75	60	72

a) Approximate $\int_0^{50} v(t)$ with a Riemann sum, using the midpoints of five subintervals of equal length.

b) Using the correct units, explain the meaning of this integral.

4. Tide removes sand from a beach at a rate modeled by the function R(t), in cubic yards, per hour and *t* is measured in hours, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. Interpret the meaning of $\int_0^6 R(t)dt$.

**calc. FR 5. The rate of fuel consumption (in gallons per minute) recorded during a plane flight is given by a twice differentiable and strictly increasing function R of time *t*.



t	R(t)
0	20
30	30
40	40
50	55
70	65
90	70

a. Approximate the value of the total fuel consumption, using a left Riemann sum with the 5 subintervals indicated in the table above.

b. Is this numerical approximation less that the value of the exact area? Explain your reasoning.

c. For $0 < b \leq 90$, explain the meaning of $\int_0^b R(t)dt$ in terms of fuel consumption for the plane. Indicate units.

6. The rate at which oil flow out of a pipe, in gallons per hour, is given by a differentiable function R(t), for $0 \leq t \leq 15$ hours. Use a left Riemann sum with subintervals indicated in the table of R(t) below to approximate the total amount of oil that flowed out of the pipe for $3 \leq t \leq 12$ hours.

t (sec)	0	3	6	9	12	15
v(t) ft/sec	9.6	10.4	10.8	11.2	11.4	11.3

7. In the linear piecewise function *f* to the right, *f* has the given points and is defined for $-2 \leq x \leq 3$. What is the value of $\int_{-2}^{3} f(x)dx$?