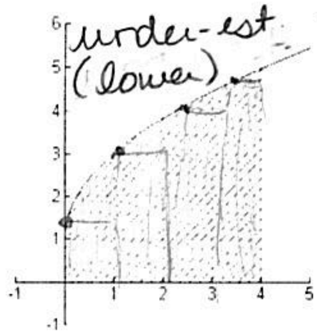
AP Calc BC Worksheet 5.2 and 5.3 ☆ Calc ok

D1
10/17

1) Bound the area of the shaded region by approximating the left and right sums. Use rectangles of width 1.

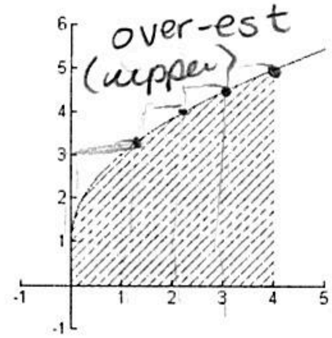
Left Sum




LRAM
$1(1.3+3+4.1+4.5)$
$\boxed{\approx 12.9}$

RRAM
$1(4.8+4.5+4.1+3)$
$\boxed{=16.3}$

Right Sum

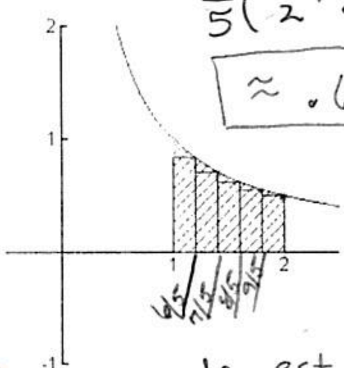



2) Use the upper and lower sums to approximate the area of the region using the given number of subintervals of equal width. $y=\frac{1}{x}$

$\Delta x = \frac{1}{5}$ RRAM $\frac{1}{5}\left(f(2)+f(1\tfrac{4}{5})+f(1\tfrac{3}{5})+f(1\tfrac{2}{5})+f(1\tfrac{1}{5})\right)$

$\frac{1}{5}\left(\frac{1}{2}+\frac{5}{9}+\frac{5}{8}+\frac{5}{7}+\frac{5}{6}\right)$

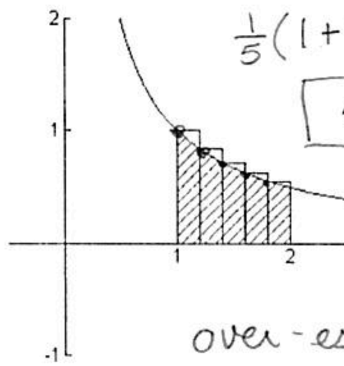
$\boxed{\approx .6455}$



under-est
(lower sum)

$\frac{1}{5}\left(1+\frac{5}{6}+\frac{5}{7}+\frac{5}{8}+\frac{5}{9}\right)$

$\boxed{\approx .7456}$

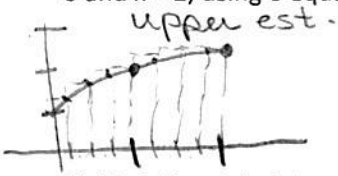


over-est. (upper sum)

LRAM $=\frac{1}{5}\left(f(1)+f(1\tfrac{1}{5})+f(1\tfrac{2}{5})+f(1\tfrac{3}{5})+f(1\tfrac{4}{5})\right)$

3) Find the right Reimann sum for the area of the region bounded by the graph of $y=\sqrt{x}+1$ and the x-axis between x = 0 and x = 2, using 8 equal width rectangles. $\Delta x = \frac{1}{4}$

upper est.



$\frac{1}{4}\big(f(2)+f(1.75)+f(1.5)+f(1.25)+$
$f(1)+f(.75)+f(.5)+f(.25)\big)$

$\approx 4.038$

4) Find the midpoint sum to approximate the area of the region bounded by $f(x)=x^2+4x$ over the interval [0, 4] using 4 equal width rectangles. $\Delta x = \frac{4-0}{4}=1$

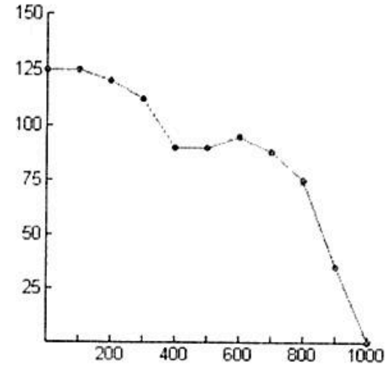
| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 0 | 5 | 12 | 21 | 32 |

$1\left(f(.5)+f(1.5)+f(2.5)+f(3.5)\right)$

$=2.25+8.25+16.25+26.25$

$\boxed{=53}$

5. Use the graph and table for the following questions:




| L1 | L2 |
|---|---|
| 0 | 125 |
| 100 | 125 |
| 200 | 120 |
| 300 | 112 |
| 400 | 90 |
| 500 | 90 |
| 600 | 95 |
| 700 | 88 |
| 800 | 75 |
| 900 | 35 |
| 1000 | 0 |

a) Estimate the area using right Riemann sums with 5 equal width rectangles. $\Delta x = 200$

$200(0+75+95+90+120) = \boxed{76{,}000}$

RRAM

b) Estimate the area using left Riemann sums with 5 equal width rectangles.

$200(125+120+90+95+75) =$

LRAM $\boxed{101{,}000}$

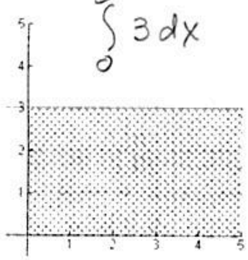
c) Estimate the area using midpoint Riemann sums with 5 subintervals of equal length.

$200(125+112+90+88+35) = \boxed{90{,}000}$

**Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)**

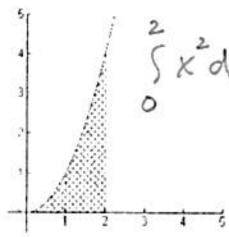
6. $f(x) = 3$

$\int_0^5 3\,dx$



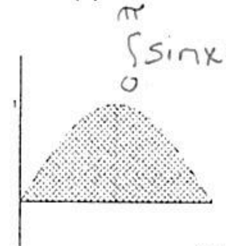
7. $f(x) = x^2$

$\int_0^2 x^2\,dx$



8. $f(x) = \sin x$

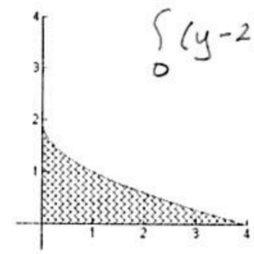
$\int_0^{\pi} \sin x\,dx$



π

9. $f(y) = (y-2)^2$

$\int_0^4 (y-2)^2\,dy$



**Sketch the region whose area is given by the definite integral. Then use the geometric formula to evaluate the integral (a > 0, r > 0)**

10. $\int_0^4 x\,dx$

$\frac{1}{2}(4\cdot 4) = \boxed{8}$

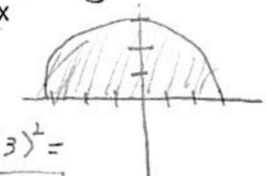
11. $\int_0^8 (8-x)\,dx$

$\frac{1}{2}(8\cdot 8) = \boxed{32}$

12. $\int_{-3}^{3} \sqrt{9-x^2}\,dx$

$y^2 = 9-x^2$

$x^2+y^2=9$



$\frac{1}{2}\pi(3)^2 = \boxed{\frac{9\pi}{2}}$

**For problems #13 – 15, evaluate the integral using the following values.**

$\int_2^4 x^3\,dx = 60$ $\qquad$ $\int_2^4 x\,dx = 6$ $\qquad$ $\int_2^4 dx = 2$

13. $\int_2^2 x^3\,dx$

$\boxed{0}$

14. $\int_2^4 (x-8)\,dx = \int_2^4 x\,dx - \int_2^4 8\,dx$

$6 - \int_2^4 8\,dx = 6-16$

$= \boxed{-10}$

15. $\int_2^4 (6+2x-x^3)\,dx = \int_2^4 6\,dx + 2\int_2^4 x\,dx - \int_2^4 x^3\,dx$

$= \int_2^4 6\,dx + 12 - 60$

$= \int_2^4 6\,dx - 48 = 12-48 = \boxed{-36}$

16. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$

a. $\int_2^6 [g(x)-f(x)]\,dx$

$-2-10$

$\boxed{-12}$

b. $\int_2^6 2g(x)\,dx$

$= 2\int_2^6 g(x)\,dx$

$\boxed{-4}$

c. $\int_2^6 3f(x)\,dx$

$= 3\int_2^6 f(x)\,dx$

$\boxed{30}$

17. Use the table of values to estimate $\int_0^6 f(x)\,dx$. Use three subintervals and the (a) left endpoints, (b) right endpoints, and (c) midpoints. If f is an increasing function, how does each estimate compare with the actual value? $\Delta x = 2$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| f(x) | -6 | 0 | 8 | 18 | 30 | 50 | 80 |

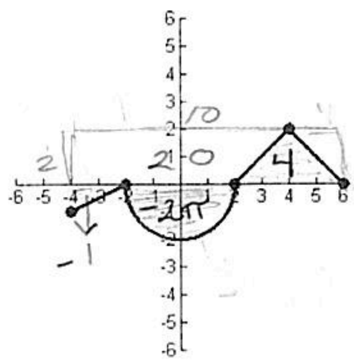
LRAM $2(-6+8+30) = 64$

RRAM $2(80+30+8) = 236$

Midpoint $2(50+18+0)$ $136$

If f(x) is increasing, LRAM is under-est & RRAM over-est. Midpoint much more accurate.

18. The graph of f consists of line segments and a semicircle. Evaluate each definite integral by using geometric formulas.



a) $\int_0^2 f(x)\,dx$ $-\pi$

b) $\int_2^6 f(x)\,dx$ $4$

c) $\int_{-4}^2 f(x)\,dx$ $-1-2\pi$

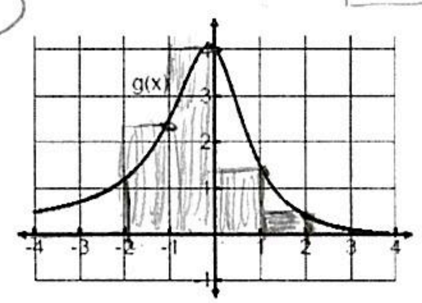
d) $\int_{-4}^6 f(x)\,dx$ $3-2\pi$

e) $\int_{-4}^6 |f(x)|\,dx$ $5+2\pi$

f) $\int_{-4}^6 [f(x)+2]\,dx$ $20$

$3-2\pi+16 = \boxed{19-2\pi}$

From NC #48, 49, 50, 51




1. Set up an estimate of the area between the curve g(x) and the x-axis using the right Riemann sums with 4 subintervals of equal width for [-2, 2]. $\Delta x = 1$

$A \approx 1(.4 + 1.4 + 4 + 2.4)$

$\approx 8.2$

2. Suppose a car has the velocities, in feet per second, represented in the table below. Find the estimate of the total distance for [0, 12] seconds using the midpoints of 3 rectangles. $\Delta x = 4$

| t (sec) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| v(t) ft/sec | 20 | 30 | 38 | 44 | 43 | 35 | 39 |

$4(30+44+35) =$ 436 feet

**calc. FR 3. In the table below, v(t) represents the velocity, in ft/sec, of a car traveling on a straight road.

| t (sec) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| v(t) ft/sec | 0 | 12 | 20 | 30 | 55 | 70 | 78 | 81 | 75 | 60 | 72 |

a) Approximate $\int_0^{50} v(t)$ with a Riemann sum, using the midpoints of five subintervals of equal length. $\Delta x = 10$

$10(12+30+70+81+60) =$ 2230 feet

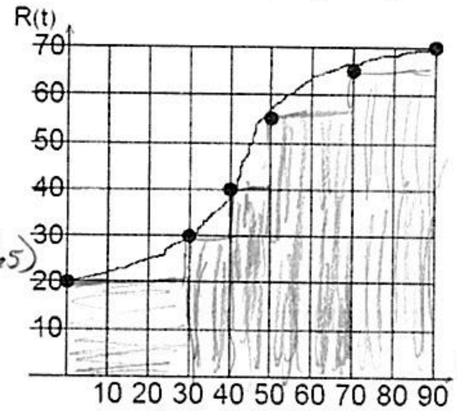
b) Using the correct units, explain the meaning of this integral.

From 0 to 50 seconds, the car traveled a distance of 2,230 ft.

4. Tide removes sand from a beach at a rate modeled by the function R(t), in cubic yards, per hour and *t* is measu

hours, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. Interpret the meaning of $\int_0^6 R(t)dt$. Amount of sand removed from 0 to 6 hrs.

**calc. FR 5. The rate of fuel consumption (in gallons per minute) recorded during a plane flight is given by a twice differentiable and strictly increasing function R of time *t*.




| t | R(t) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

a. Approximate the value of the total fuel consumption, using a left Riemann sum with the 5 subintervals indicated in the table above.

LRAM:
$30(20) + 10(30) + 10(40) + 10(55) + 20(65)$
$= 3150$ gallons

b. Is this numerical approximation less that the value of the exact area? Explain your reasoning.

Less than exact area, since the function is increasing.

c. For $0 < b \le 90$, explain the meaning of $\int_0^b R(t)dt$ in terms of fuel consumption for the plane. Indicate units.

This gives total gallons consumed from 0 to "b" minutes.

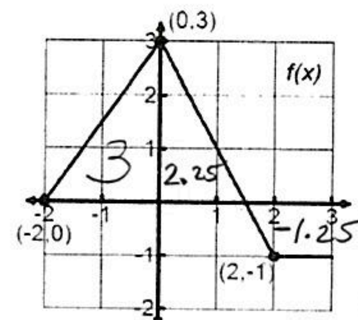
6. The rate at which oil flow out of a pipe, in gallons per hour, is given by a differentiable function R(t), for $0 \le t \le 15$ hours. Use a left Riemann sum with subintervals indicated in the table of R(t) below to approximate the total amount of oil that flowed out of the pipe for $3 \le t \le 12$ hours.

| t (sec) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| v(t) ft/sec | 9.6 | 10.4 | 10.8 | 11.2 | 11.4 | 11.3 |

$3(9.6 + 10.4 + 10.8 + 11.2 + 11.4)$
$3(53.4) = 160.2$ gallons

7. In the linear piecewise function *f* to the right, *f* has the given points and is defined for $-2 \le x \le 3$. What is the value of $\int_{-2}^{3} f(x)dx$?

$3 + 2.25 - 1.25 = \boxed{4}$




$A_{Trap} = \left(\frac{b_1 + b_2}{2}\right)h$

$\left(\frac{1 + 1.5}{2}\right)(1)$