non-calc!

D1 HW

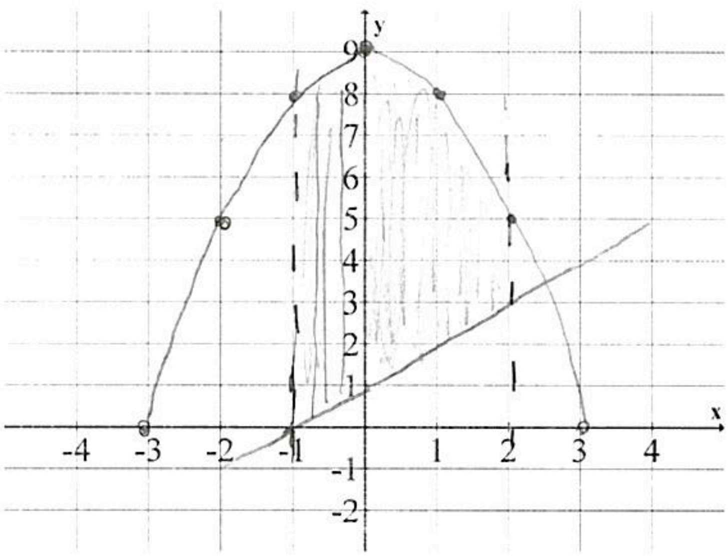
$A = \int_{x_1}^{x_2} [(\text{Top Curve}) - (\text{Bottom Curve})]\,dx$ for vertical rectangles

$A = \int_{y_1}^{y_2} [(\text{Right Curve}) - (\text{Left Curve})]\,dy$ for horizontal rectangles

For each of the following problems:

1. Sketch the region enclosed by the given curves.
2. Decide whether to integrate with respect to x or y.
3. Find the area of the region.

1. $y = x + 1 \quad y = 9 - x^2 \quad x = -1 \quad x = 2$



$\int_{-1}^{2} [(9-x^2)-(x+1)]\,dx$

$\int_{-1}^{2} (8-x-x^2)\,dx =$

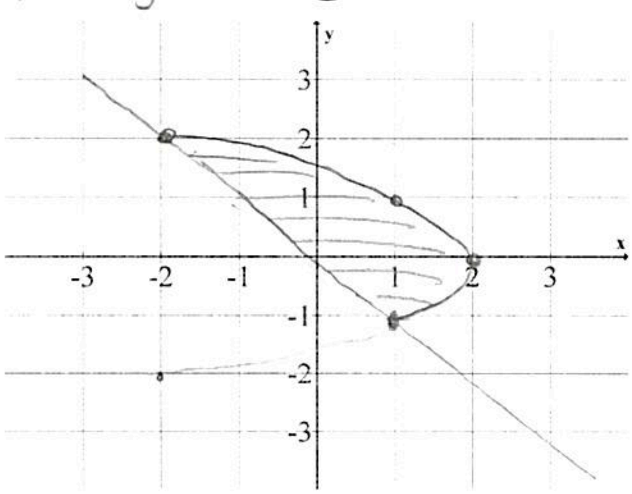
$8x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-1}^{2} =$

$\left(16 - 2 - \frac{8}{3}\right) - \left(-8 - \frac{1}{2} + \frac{1}{3}\right) =$

$\boxed{19\tfrac{1}{2}}$

2. $x + y^2 = 2 \quad y + x = 0$

$x = 2 - y^2 \quad x = -y$



$\int_{-1}^{2} [(2-y^2)-(-y)]\,dy =$

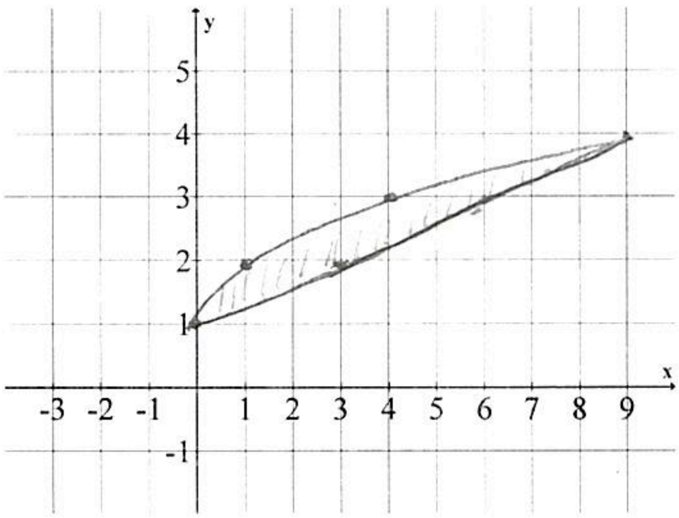
$2y - \frac{y^3}{3} + \frac{y^2}{2} \Big|_{-1}^{2} =$

$\left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$

$8 - 3\tfrac{1}{2} = \boxed{4\tfrac{1}{2}}$

$y = 1 + \frac{1}{3}x$

3. $y = 1 + \sqrt{x} \quad y = \dfrac{3+x}{3}$

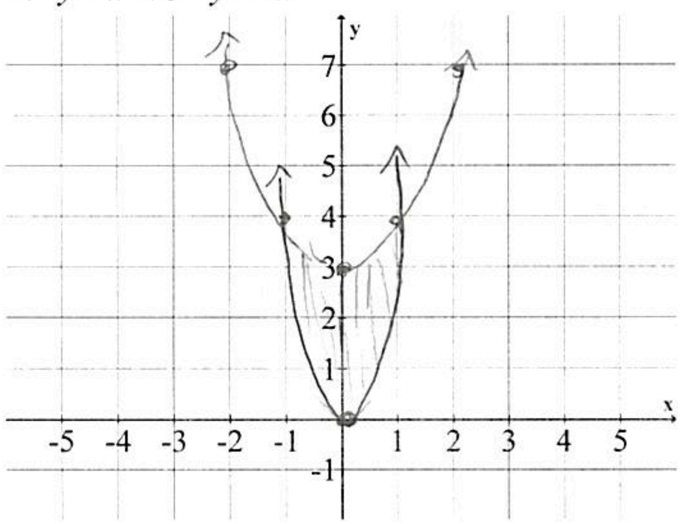


$$\int_0^9 \left[(1+\sqrt{x}) - \left(1+\frac{x}{3}\right)\right] dx$$

$$\cancel{x} + \frac{2}{3}x^{3/2} - \cancel{x} - \frac{x^2}{6}\Big|_0^9$$

$$18 - 13.5 = \boxed{4.5}$$

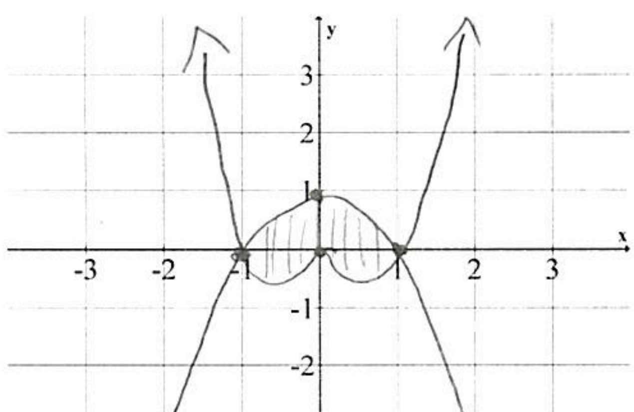
4. $y = x^2 + 3 \quad y = 4x^2$



$$\int_{-1}^{1} \left[(x^2+3) - 4x^2\right] dx$$

$$\boxed{4}$$

5. $y = x^4 - x^2 \quad y = 1 - x^2$



$$\int_{-1}^{1} \left[(1-x^2) - (x^4-x^2)\right] dx$$

$$\int_{-1}^{1} (1-x^4)\, dx =$$

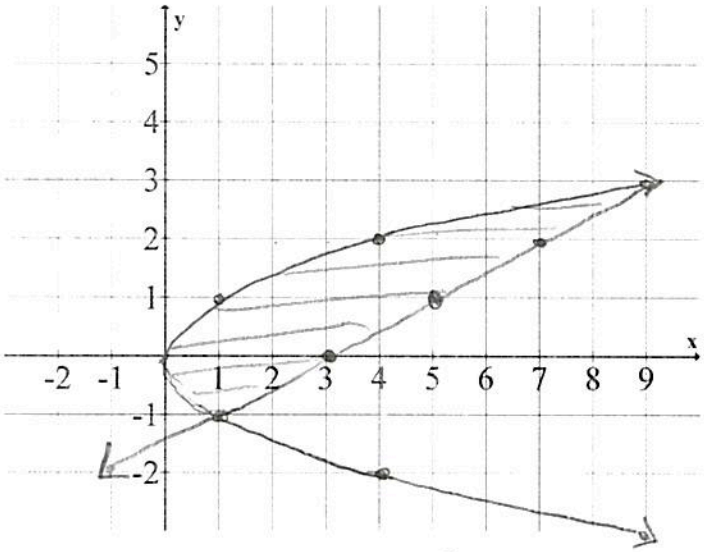
$$x - \frac{x^5}{5}\Big|_{-1}^{1} =$$

$$\left(1-\frac{1}{5}\right) - \left(-1+\frac{1}{5}\right) =$$

$$2 - \frac{2}{5} = \boxed{1\frac{3}{5}}$$

$x = 2y+3$

6. $y^2 = x \quad x - 2y = 3$



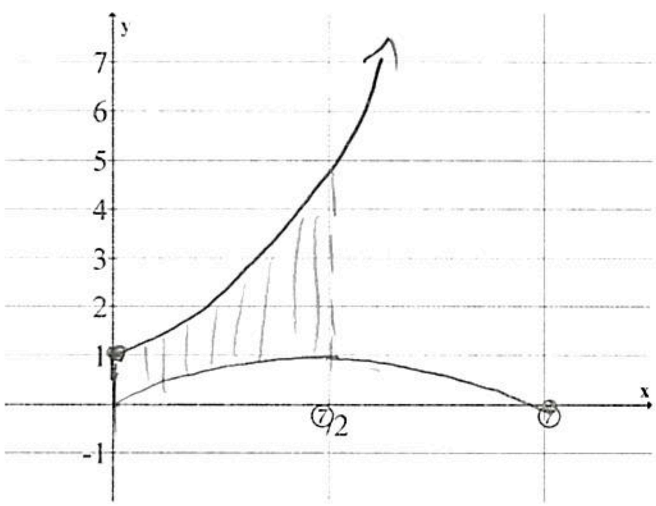
$$\int_{-1}^{3}\left[(2y+3)-(y^2)\right]dy =$$

$$y^2+3y-\frac{y^3}{3}\Big|_{-1}^{3} =$$

$$(\cancel{9}+9-\cancel{9})-\left(1-3+\frac{1}{3}\right) =$$

$$\boxed{10\frac{2}{3}}$$

7. $y = \sin x \quad y = e^x \quad x = 0 \quad x = \frac{\pi}{2}$



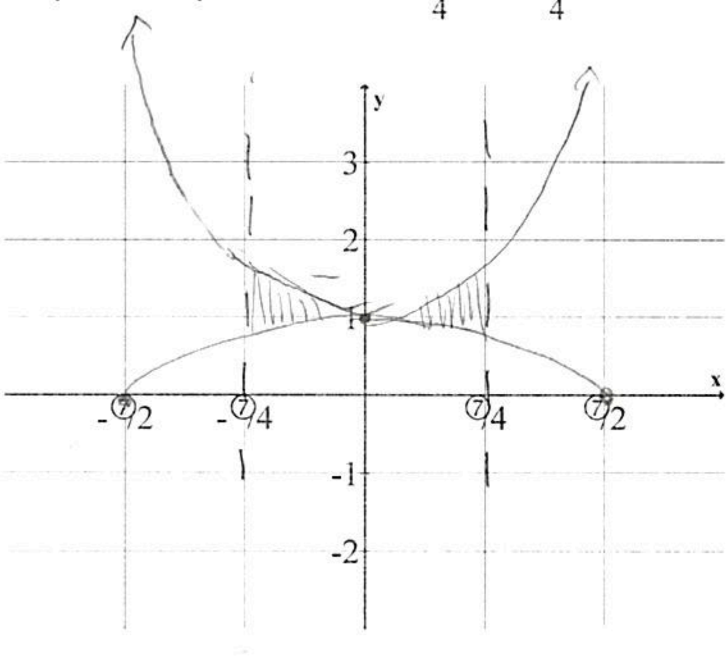
$$\int_{0}^{\pi/2}(e^x-\sin x)\,dx =$$

$$e^x+\cos x\Big|_{0}^{\pi/2}$$

$$(e^{\pi/2}+\cos\pi/2)-(e^0+\cos 0)$$

$$e^{\pi/2}-(1+1) = \boxed{e^{\pi/2}-2}$$

8. $y = \cos x \quad y = \sec^2 x \quad x = \frac{-\pi}{4} \quad x = \frac{\pi}{4}$



$$\int_{-\pi/4}^{\pi/4}(\sec^2 x-\cos x)\,dx =$$

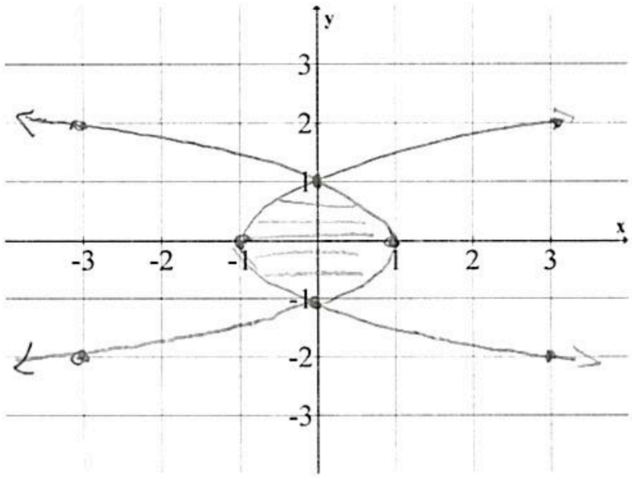
$$\tan x-\sin x\Big|_{-\pi/4}^{\pi/4} =$$

$$(\tan\pi/4-\sin\pi/4)-(\tan-\pi/4-\sin-\pi/4)$$

$$\left(1-\frac{\sqrt{2}}{2}\right)-\left(-1+\frac{\sqrt{2}}{2}\right)$$

$$\boxed{2-\sqrt{2}}$$

9. $x = 1 - y^2 \quad x = y^2 - 1$



$$\int_{-1}^{1} \left[(1-y^2)-(y^2-1)\right]dy =$$

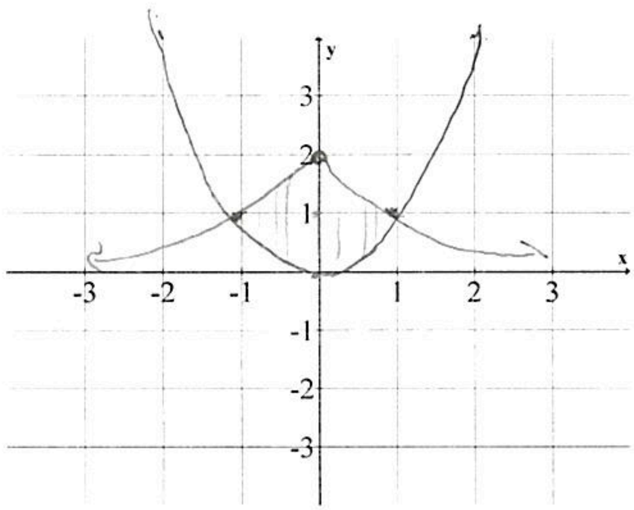
$$\int_{-1}^{1}(-2y^2+2)\,dy =$$

$$-\frac{2}{3}y^3+2y \Big|_{-1}^{1} =$$

$$\left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right)$$

$$4-\frac{4}{3} = \boxed{2\tfrac{2}{3}}$$

10. $y = x^2 \qquad y = \dfrac{2}{x^2+1}$



$$\int_{-1}^{1}\left[\left(\frac{2}{x^2+1}\right)-x^2\right]dx =$$

$$2\int_{-1}^{1}\frac{1}{x^2+1}-x^2)\,dx = 2\left[\arctan x-\frac{x^3}{3}\right|_{-1}^{1}$$

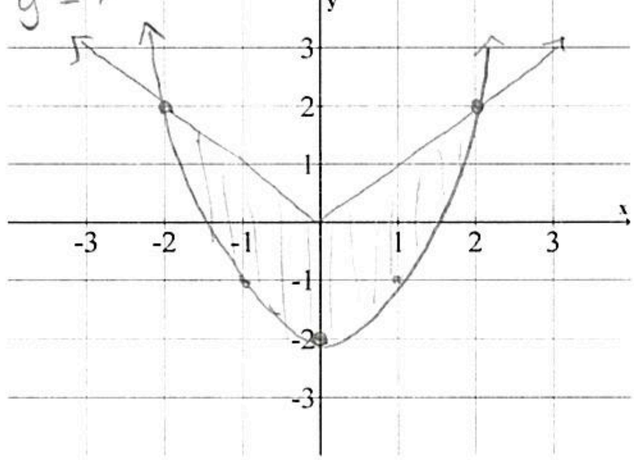
$$= 2\left[\left(\frac{\pi}{4}-\frac{1}{3}\right)-\left(-\frac{\pi}{4}+\frac{1}{3}\right)\right]$$

$$= 2\left(\frac{\pi}{2}-\frac{2}{3}\right) = \boxed{\pi-4/3}$$

11. $y = |x| \qquad y = x^2 - 2$

(-2, 0) $y = -x$

(0, 2) $y = x$



$$\int_{-2}^{0}\left[-x-(x^2-2)\right]dx + \int_{0}^{2}\left[x-(x^2-2)\right]dx$$

$$-\frac{x^2}{2}-\frac{x^3}{3}+2x\Big|_{-2}^{0} + \frac{x^2}{2}-\frac{x^3}{3}+2x\Big|_{0}^{2}$$

$$0-\left(-2+\frac{8}{3}-4\right)+2-\frac{8}{3}+4-0$$

$$-(-6+8/3)+6-8/3$$

$$12-16/3 = \boxed{6\,2/3}$$