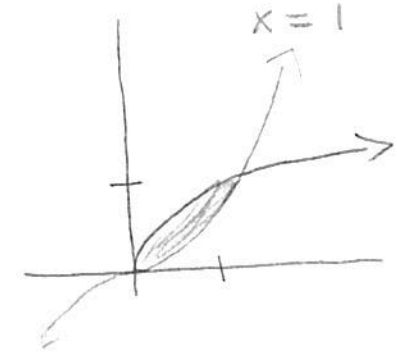
AP Problems Area

(1) Let R be the region enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$.

$x^3 = \sqrt{x}$ @ $x = 0$ & $x = 1$



(a) Find the area of R.

$$\int_0^1 (x^{1/2} - x^3)\,dx = \frac{2}{3}x^{3/2} - \frac{x^4}{4}\Big|_0^1$$

$$\frac{2}{3} - \frac{1}{4} = \boxed{\frac{5}{12}}$$

(2) Given the parabola $y = x^2 - 2x + 3$.

(a) Find an equation for the line l which contains the point $(2,3)$ and is perpendicular to the line tangent to the parabola at $(2,3)$.

(b) Find the area of that part of the first quadrant which lines below <u>both</u> the line l and the parabola.

a) $y' = 2x - 2$

$y'(2) = 2 \rightarrow m$

$\perp m = -\frac{1}{2}$

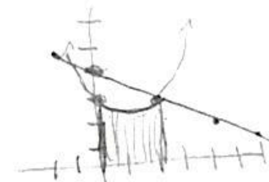
$y - 3 = -\frac{1}{2}(x-2)$

$\star\ y = -\frac{1}{2}x + 4$

b) $\int_0^2 (x^2 - 2x + 3)\,dx =$

$\frac{x^3}{3} - x^2 + 3x \Big|_0^2 =$

$\frac{8}{3} - 4 + 6 = \boxed{14/3}$



(3) Let R be the region between the graphs of $y = 1 + \sin(\pi x)$ and $y = x^2$ from $x = 0$ to $x = 1$.

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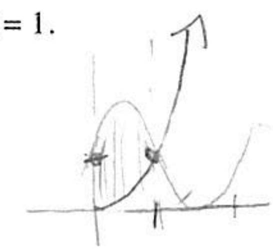


(a) Find the area of R.

$u = \pi x$

$du = \pi dx$

$dx = \frac{du}{\pi}$

$$A = \int_0^1 (1 + \sin \pi x - x^2)\,dx = x - \frac{1}{\pi}\cos(\pi x) - \frac{x^3}{3}\Big|_0^1$$

$$= \left(1 + \frac{1}{\pi} - \frac{1}{3}\right) - \left(0 - \frac{1}{\pi} - 0\right)$$

$$= \boxed{\frac{2}{3} + \frac{2}{\pi}}$$

(4) Let R be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x+4}$, the line $y = 2x$, and the <u>y-axis</u>.

$x = 0$

Find Limits: $\sqrt{6x+4} = 2x$

$6x + 4 = 4x^2$

$4x^2 - 6x - 4 = 0$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$\cancel{x = -\frac{1}{2}}$ $x = 2$

(a) Find the area of R.

$u = 6x + 4$

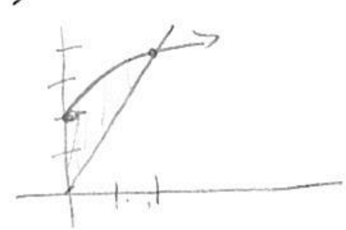
$du = 6dx$

$dx = \frac{du}{6}$

$$\int_0^2 ((6x+4)^{1/2} - 2x)\,dx =$$

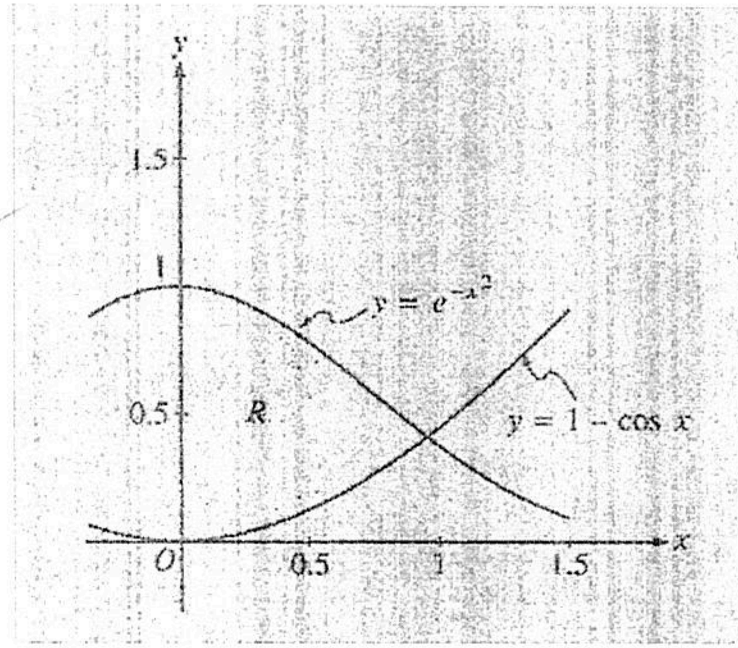
$$\frac{1}{6}\int_0^2 u^{1/2}\,du - \int_0^2 2x\,dx = \frac{1}{9}(6x+4)^{3/2} - x^2 \Big|_0^2$$

$$\left(\frac{64}{9} - 4\right) - \frac{8}{9} = \boxed{\frac{20}{9}}$$



(5)

Calculator



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

$$\int_0^{.9419} \left(e^{-x^2} - (1-\cos x)\right)dx \approx \boxed{.591}$$

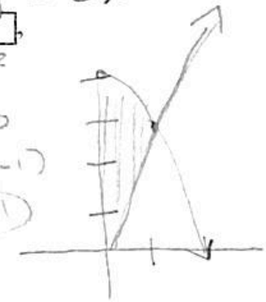
(6) Let R be the region in the first quadrant enclosed by the graphs of $\boxed{y=4-x^2}$, $\boxed{y} = 3x$, and the y-axis. $\rightarrow x=0$

$3x = 4-x^2$

$x^2+3x-4=0$

$(x+4)(x-1)$

$x=1$



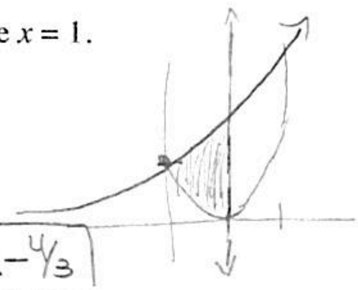
(a) Find the area of region R.

$$\int_0^1 (4-x^2-3x)\,dx = 4x - \frac{x^3}{3} - \frac{3}{2}x^2 \Big|_0^1$$

$$4-\frac{1}{3}-\frac{3}{2} = \boxed{\frac{13}{6} = 2\frac{1}{6}}$$

(7) Let R be the region enclosed by the graphs of $y=e^x$, $y=(x-1)^2$, and the line $x=1$.

$y = x^2-2x+1$

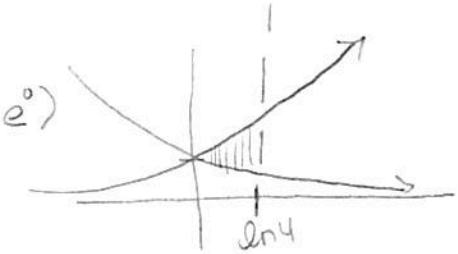


(a) Find the area of R.

$$\int_0^1 \left(e^x - (x^2-2x+1)\right)dx =$$

$$e^x - \frac{x^3}{3} + x^2 - x \Big|_0^1 = \left(e - \frac{1}{3} + \cancel{1} - \cancel{1}\right) - (e^0 - 0) = \boxed{e - 4/3}$$

(8) Let R be the region enclosed by the graphs of $\boxed{y=e^{-x}}$, $\boxed{y=e^x}$, and $\boxed{x=\ln 4}$.



(a) Find the area of R by setting up and evaluating a definite integral.

$$\int_0^{\ln 4} (e^x - e^{-x})\,dx = e^x - (-1)e^{-x} \Big|_0^{\ln 4} = (e^{\ln 4} + e^{-\ln 4}) - (e^0 + e^0)$$

$$= 4 + e^{\ln 4^{-1}} - 2$$

$$= 2 + \frac{1}{4} = \boxed{2\,1/4}$$

(9) Given the function f defined by all real numbers x by $\boxed{f(x) = e^{x/2}}$.

(a) Find the area of the region R bounded by the line $\boxed{y=e}$, the graph of f, and the y-axis.

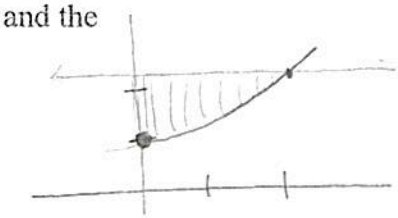
$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$dx = 2du$

$$\int_0^2 \left(e - e^{\frac{1}{2}x}\right)dx =$$

$$ex - 2e^{1/2 x} \Big|_0^2 = (2e - 2e) - (-2) = \boxed{2}$$



(10) Let R be the region in the first quadrant that is enclosed by the graph of $\boxed{y=\tan x}$, the x-axis, and the line $\boxed{x=\pi/3}$.

$$\int_0^{\pi/3} \tan x\,dx =$$

(a) Find the area of R.

$$-\ln|\cos x| \Big|_0^{\pi/3} = -\ln\cos\pi/3 + \ln\cos 0$$

$$= -\ln\tfrac{1}{2} + \ln 1$$

$$= \boxed{\ln 2}$$