Irén Manvelova
AP CALCULUS BC
2B

DIRECT COMPARISON

N TH TERM TEST

LIMIT COMPARISON

ROOT TEST

$$\sum_{K=1}^{\infty} U_K$$

$$L = \lim_{K \to \infty} U_K^{1/K}$$

$L > 1 \rightarrow$ DIVERGES

$L < 1 \rightarrow$ CONVERGES

$L = 1$ INCONCLUSIVE

EX. $\sum_{K=1}^{\infty} K^K$

$$\lim_{K \to \infty} \left[K^K\right]^{1/K} = \lim_{K \to \infty} K^1 = \infty$$

Diverges

GEOMETRIC

INTEGRAL TEST

$$\sum_{K=1}^{\infty} \frac{K^3}{3^K}$$

$$\lim_{K \to \infty} \left[\frac{K^3}{3^K}\right]^{1/K} = \lim_{K \to \infty} \frac{K^{3/K}}{3^1} =$$

converges

P-SERIES

RATIO TEST

ALTERNATING

EX. $\sum_{n=1}^{\infty}\left[\frac{2n+3}{3n-5}\right]$ DIVERGES B/C

$\lim_{n\to\infty}\left[\frac{2n+3}{3n-5}\right] = \frac{2}{3} \neq 0$

$N^\circ = D^\circ \to$ CONVERGE

$N^\circ < D^\circ \to$ LIMIT $= 0 \to$ CONVERGE

$N^\circ > D^\circ \to$ DNE

IF $\sum_{n=1} a_n$ CONVERGES $\to \lim_{n\to\infty} a_n = 0$

IF $\lim_{n\to\infty} a_n \neq 0 \to \sum_{n=1}^{\infty} a_n$ DIVERGES

*** DOES NOT MEAN THAT IF $\lim_{n\to\infty} a_n = 0$ THAT THE SERIES CONVERGES

[TEST ONLY FOR DIVERGENCE]

$b_n > a_n$

$\sum_{n=1}^{\infty} b_n$ CONVERGES $\to \sum_{n=1}^{\infty} a_n$ CONVERGES

$\sum_{n=1}^{\infty} a_n$ DIVERGES $\to \sum_{n=1}^{\infty} b_n$ DIVERGES

*COMMON USE OF GEOMETRIC AND P-SERIES

EX. $\sum_{n=1}^{\infty}\left[\frac{1}{n^3+1}\right]$ $\quad \sum_{n=1}^{\infty} a_n$ CONVERGE

$\sum_{n=1}^{\infty}\frac{1}{n^3} > \sum_{n=1}^{\infty}\frac{1}{n^3+1}$

$p = 3 > 1$

$\downarrow$

CONVERGES

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a is the first term

r = ratio

SUM $= \frac{a}{1-r}$

EX. $0.\overline{8}$

$\frac{8}{10} + \frac{8}{100} + \frac{8}{1000} \ldots$ $\quad$ sum $= \frac{8}{9}$

$r = 1/10$ $\quad a = 8/10 = 4/5$

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \ldots ar^n$

$a \neq 0$

DIVERGES $|r| \geq 1$

CONVERGES $|r| < 1$ to the sum

$\downarrow$

$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, $0 < |r| < 1$

*positive

$\lim_{n\to\infty} \frac{{}^{*}a_n}{b_n} = L$

• IF LIMIT IS 0 AND $\sum b_n$ CONVERGES, THEN $\sum a_n$ CONVERGES

• IF LIMIT IS ∞ AND $\sum b_n$ DIVERGES, THEN $\sum a_n$ DIVERGES

$\sum_{n=1}^{\infty} \sin\frac{1}{n}$ $\quad b_n = 1/n \to$ Harmonic Diverges

$\lim_{n\to\infty} \frac{\sin 1/n}{1/n} = \frac{0}{0}$

By L'Hôpital's

$\lim_{n\to\infty} \frac{\cancel{-1/n^2}\cos 1/n}{\cancel{-1/n^2}} = 1$ $\to$ positive $\to$ Diverg

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EX. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$

$p = 2 > 1 \to$ co

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ CONVERGES

$\sum_{n=1}^{\infty}\frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots$

$p > 1 \to$ CONVERGES

$p \leq 1 \to$ DIVERGES

f MUST BE:

1. CONTINUOUS $\quad$ Derivative
2. POSITIVE
3. DECREASING FOR $x \geq 1$

*USE IMPROPER INTEGRAL

$\lim_{b\to\infty}\int_1^b f(x)\,dx$

EX. $\sum_{n=1}^{\infty}\frac{1}{n+1}$ diverges B/C

$\int_1^{\infty}\frac{dn}{n+1} = \lim_{b\to\infty}\int_1^b \frac{dn}{n+1} =$

$\lim_{b\to\infty} \ln|n+1|\Big]_1^{\infty} = \ln|\infty+1| - \ln|2| = \infty$

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① Each a_n is positive

② $|a_{n+1}| \leq |a_n|$

③ $\lim_{n\to\infty} a_n = 0$

Convergent test

EX. $\sum_{n=1}^{\infty}(-1)^{n+1}\left[\frac{2n}{4n-3}\right]$ $\quad b_n = \frac{2n}{4n-3}$

$\frac{2(n+1)}{4(n+1)-3} < \frac{2n}{4n-3}$

$\lim_{n\to\infty} b_n = \frac{1}{2}$ $\quad \sum_{n=1}^{\infty}(-1)^{n+1}\left[2n/4n-3\right]$ diverges

$\lim_{n\to\infty}\left|\frac{a_{(n+1)}}{a_n}\right| < 1 \to \sum a_n$ converges

$\lim_{n\to\infty}\left|\frac{a_{(n+1)}}{a_n}\right| > 1$ OR $= \infty \to$ DIVERGES

$\lim_{n\to\infty}\left|\frac{a_{(n+1)}}{a_n}\right| = 1 \to$ INCONCLUSIVE

EX. $\sum_{n=1}^{\infty}\left[3^n/n^2\right]$

$\left.\lim_{n\to\infty}\left|\frac{3^{n+1}}{(n+1)^2}\cdot\frac{n^2}{3^n}\right|\right\}$ SIMPLIFY

$= 1 \to$ INCONCLUSIVE