Ch 8 Review

① $\int \frac{x}{x^2-4} dx$ $\quad u = x^2-4$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u} du$ $\quad dx = \frac{1}{2x}$

$= \frac{1}{2}\ln|u| + C \Rightarrow \boxed{\frac{1}{2}\ln|x^2-4| + C}$

② $\int \frac{2x+9}{x^2+4} dx = \int\left(\frac{2x}{x^2+4} + \frac{9}{x^2+4}\right)dx$

$= \int \frac{2x}{x^2+4} dx + \int \frac{9}{x^2+4} dx$

$\downarrow \qquad \downarrow$

$u = x^2+4$ $\quad$ this is arctan!

$du = 2x\,dx$

$= \int \frac{1}{u} du + 9\int \frac{1}{x^2+4} dx \rightarrow a = 2$

$\boxed{\ln(x^2+4) + \frac{9}{2}\arctan\left(\frac{x}{2}\right) + C}$

(always +)

③ $\int \sin^3 x \cos^2 x\, dx =$

$\int \sin x \sin^2 x \cos^2 x\, dx =$

$\int \sin x(1-\cos^2 x)\cos^2 x\, dx =$

$\int \sin x(\cos^2 x - \cos^4 x)\, dx =$ $\quad u = \cos x$

$du = -\sin x\, dx$

$-\int(u^2-u^4)du =$

$-\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C \Rightarrow \boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$

(4) $\int \cos^3\theta \sin^8\theta \, d\theta =$

$\int \cos\theta \cos^2\theta \sin^8\theta \, d\theta =$

$\int \cos\theta (1-\sin^2\theta)\sin^8\theta \, d\theta =$

$\int \cos\theta (\sin^8\theta - \sin^{10}\theta) d\theta$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$\int (u^8 - u^{10}) du \leftarrow d\theta = du/\cos\theta$

$\frac{1}{9}u^9 - \frac{1}{11}u^{11} + C \Rightarrow \boxed{\frac{1}{9}\sin^9\theta - \frac{1}{11}\sin^{11}\theta + C}$

(5) $\int x e^{-12x} dx$ int. by pts

$u = x \longrightarrow v = -\frac{1}{12}e^{-12x}$

$du = dx \quad dv = e^{-12x}$

$\int x e^{-12x} dx = -\frac{1}{12}x e^{-12x} - \int -\frac{1}{12}e^{-12x} dx$

$= \boxed{-\frac{1}{12}xe^{-12x} - \frac{1}{144}e^{-12x} + C}$

(6) $\int \frac{4x+4}{x^2-2x-15} dx$ (Partial Fraction decomp)

$= \int \frac{4x+4}{(x-5)(x+3)} = \int \left(\frac{A}{(x-5)} + \frac{B}{(x+3)}\right) dx = \int \frac{A(x+3)+B(x-5)}{(x-5)(x+3)}$

$4x+4 = A(x+3) + B(x-5)$

Let $x = -3 \rightarrow -8 = 0 + B(-8)$ $B=1$

let $x = 5 \rightarrow 24 = 8A$ $A=3$

$\therefore \int \frac{4x+4}{x^2-2x-15} dx = \int \frac{3}{x-5} dx + \int \frac{1}{x+3} dx$

$= \boxed{3\ln|x-5| + \ln|x+3| + C}$

⑦ $\int \frac{dx}{x+x^{-1}} = \int \frac{1}{x+\frac{1}{x}}\,dx = \int \frac{1}{\frac{x^2+1}{x}}\,dx$

$= \int \frac{x}{x^2+1}\,dx$ $\quad u = x^2+1 \quad du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \boxed{\frac{1}{2}\ln(x^2+1)+C}$

always + so NO ABS Val needed

⑧ $\int \frac{d\alpha}{\cos^4\alpha}\,d\alpha = \int \sec^4\alpha\,d\alpha$

$= \int \sec^2\alpha(1+\tan^2\alpha)\,d\alpha$

$= \int(\sec^2\alpha + \sec^2\alpha\tan^2\alpha)\,d\alpha = \int \sec^2\alpha + \int \sec^2\alpha\tan^2\alpha$

$= \tan\alpha + \int \sec^2\alpha\tan^2\alpha\,d\alpha$ $\quad u = \tan\alpha$

$du = \sec^2\alpha\,d\alpha$

$= \tan\alpha + \int u^2\,du$

$= \boxed{\tan\alpha + \frac{1}{3}\tan^3\alpha + C}$

OR $\int(1+u^2)\,du$
$= u + \frac{u^3}{3}$

⑨ $\int \sin^2\theta \sin 2\theta\,d\theta \Rightarrow \sin 2\theta = 2\sin\theta\cos\theta$

$= \int \sin^2\theta(2\sin\theta\cos\theta)\,d\theta$

$= 2\int \sin^3\theta\cos\theta\,d\theta$ $\quad u = \sin\theta \quad du = \cos\theta\,d\theta$

$= 2\int u^3\,du = 2(\frac{1}{4}u^4)+C$

$= \boxed{\frac{1}{2}\sin^4\theta + C}$

(10) $\int \frac{dx}{x^2+x-12}$ Partial Fractions $\Rightarrow \int \left(\frac{A}{x+4} + \frac{B}{x-3}\right) dx$

$1 = A(x-3) + B(x+4)$

$x = 3 \Rightarrow \quad 1 = 7B \rightarrow B = 1/7$

$x = -4 \Rightarrow \quad 1 = -7A \rightarrow A = -1/7$

$$\int \frac{dx}{x^2+x-12} = -1/7 \int \frac{1}{x+4} dx + 1/7 \int \frac{1}{x-3} dx$$

$$= \boxed{-\frac{1}{7} \ln|x+4| + 1/7 \ln|x-3| + C}$$

(11) $\int (\sec\theta + \tan\theta)^2 d\theta$

$= \int (\sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta) d\theta \rightarrow \tan^2\theta = \sec^2\theta - 1$

$= \int (\underline{\underline{\sec^2\theta}} + 2\sec\theta\tan\theta + \underline{\underline{\sec^2\theta}} - 1) d\theta$

$= 2\int \sec^2\theta \, d\theta + 2\int \sec\theta\tan\theta \, d\theta - \int d\theta$

$= \boxed{2\tan\theta + 2\sec\theta - \theta + C}$

(12) $\int e^x \sin x \, dx$ Int by Parts $\quad u = e^x \quad v = -\cos x$

$du = e^x dx \quad dv = \sin x$

$\int e^x \sin x dx = -e^x \cos x + \int (+e^x \cos x) dx$

$u = e^x \rightarrow v = \sin x$

$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx \quad du = e^x \quad dv = \cos x \, dx$

$\frac{2\int e^x \sin x dx}{2} = \frac{-e^x \cos x + e^x \sin x}{2}$

$$\int e^x \sin x \, dx = \boxed{\begin{array}{c} \frac{1}{2}(e^x \sin x - e^x \cos x) + C \\ \text{OR} \\ \frac{1}{2} e^x (\sin x - \cos x) + C \end{array}}$$

(13) $\int x \ln x \, dx$ Int. by Parts

$u = \ln x \rightarrow v = \frac{x^2}{2}$

$du = \frac{1}{x}dx \swarrow dv = x$

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2\left(\frac{1}{x}dx\right)$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$$

$$= \boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$$

(14) $\int x^3 e^x dx =$

Tabular:

sign	u	dv
+	x^3	e^x
−	$3x^2$	e^x
+	$6x$	e^x
−	6	e^x
+	0	e^x

$$\boxed{x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C}$$

(15) $\int \sin^4 \theta \, d\theta$

$= \int \sin^2\theta \sin^2\theta \, d\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$= \int \frac{1}{2}(1 - \cos 2\theta) \cdot \frac{1}{2}(1 - \cos 2\theta) \, d\theta$

$= \frac{1}{4}\int (1 - \cos 2\theta)^2 \, d\theta$

$= \frac{1}{4}\int (1 - 2\cos 2\theta + \cos^2 2\theta) \, d\theta \Rightarrow \cos^2 2\theta = \frac{1}{2}(1 + \cos 4\theta)$

$= \frac{1}{4}\int (1 - 2\cos 2\theta + \frac{1}{2} + \frac{1}{2}\cos 4\theta) \, d\theta$

$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right) d\theta$

$u = 2\theta$ $\quad$ $u = 4\theta$

$du = 2d\theta$ $\quad$ $du = 4\,d\theta$

$= \frac{3}{8}\int d\theta - \frac{1}{4}\int \cos u \, du + \frac{1}{32}\int \cos u \, du$

$= \boxed{\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta + C}$

(16) $\int_4^{\infty} \frac{1}{x^{2/3}}\,dx = \lim_{b\to\infty} \int_4^b x^{-2/3}\,dx$

$= \lim_{b\to\infty} \left[3x^{1/3} \right]_4^b = \lim_{b\to\infty} \left[3\sqrt{b} - 3\sqrt[3]{4} \right] = \boxed{\infty \;\therefore \text{ diverges}}$

(17) $\int_0^4 \frac{dx}{(x-2)^2}$ undef@ $x=2$, so split into 2 integrals

$u = x-2$

$u = dx$

$\int u^{-2}du)$

$\lim_{b\to 2} \int_0^b \frac{dx}{(x-2)^2} + \lim_{a\to 2} \int_a^4 \frac{dx}{(x-2)^2} =$

$\lim_{b\to 2} \left[\frac{-1}{(x-2)} \right]_0^b + \lim_{a\to 2} \left[\frac{-1}{(x-2)} \right]_a^4 =$

$\lim_{b\to 2} \left[\frac{-1}{b-2} - \frac{-1}{-2} \right] + \lim_{a\to 2} \left[\frac{-1}{2} - \frac{-1}{a-2} \right]$

$(\infty - 1/2) + (-1/2 + \infty) = \boxed{\infty \;\therefore \text{ diverges}}$

(18) $\int_0^{\infty} \frac{dx}{(x+2)^2} = \lim_{b\to\infty} \int_0^b \frac{dx}{(x+2)^2}$

$= \lim_{b\to\infty} \left[\frac{-1}{x+2} \right]_0^{\infty} = \lim_{b\to\infty} \left(\frac{-1}{b+2} - \frac{-1}{2} \right) = 0 + \frac{1}{2} = \boxed{\frac{1}{2}}$

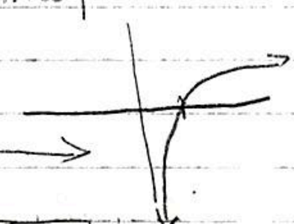
converges

(19) $\int_0^{\pi/2} \cot\theta\, d\theta \to \cot(0)$ is undefined, so:

$\lim_{a\to 0} \int_a^{\pi/2} \cot\theta\, d\theta = \lim_{a\to 0} \left[\ln|\sin\theta| \right]_a^{\pi/2}$

$= \lim_{a\to 0} \left(\ln(\sin \pi/2) - \ln|\sin a| \right.$

$= \ln 1 - \ln 0 \longrightarrow$



$0 - (-\infty) = \boxed{\infty \text{ diverges}}$

(20) $\lim_{x \to 1} \frac{x^3-1}{x-1}$ LH Rule since $\frac{0}{0}$

$= \lim_{x \to 1} \frac{3x^2}{1} = \boxed{3}$

(21) $\lim_{x \to \pi/2} \frac{\cos^2 x}{\sin x - 1}$ LH Rule since $\frac{0}{0}$

$= \lim_{x \to \pi/2} \frac{-2\cos x \sin x}{\cos x} = \lim_{x \to \pi/2} -2\sin x = \boxed{-2}$

(22) $\lim_{x \to 0^+} x \ln x \Rightarrow \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$ → LH Rule sine $\frac{-\infty}{\infty}$

$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} -x = \boxed{0}$

(23) $\lim_{x \to 0} \frac{e^x - x - 1}{\cos x - 1}$ → LH Rule since $\frac{0}{0}$

$= \lim_{x \to 0} \frac{e^x - 1}{-\sin x}$ → LH Rule since $\frac{0}{0}$

$= \lim_{x \to 0} \frac{e^x}{-\cos x} = \frac{1}{-1} = \boxed{-1}$

(24) $\lim_{x \to 1} \frac{x^2+1}{2x+1} = \boxed{2/3}$ ☺ (Thought you could use a break!)

(25) $\lim_{x \to 0^+} x^x$ → re-write as:

$\lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x}$ $\{$ on #25, we know $\lim_{x \to 0^+} x \ln x = 0$,

$\therefore \lim_{x \to 0^+} e^{x \ln x} = e^0 = 1$

(26) $\lim_{x\to 0} \frac{\sin 4x}{\sin 3x}$ LH Rule since $\frac{0}{0}$

$\lim_{x\to 0} \frac{4\cos 4x}{3\cos 3x} = \boxed{4/3}$

(27) $\lim_{x\to 2} \frac{e^{x^2}-e^4}{x-2}$ LH Rule since $\frac{0}{0}$

$\lim_{x\to 2} \frac{2xe^{x^2}}{1} = \boxed{4e^4}$

☆ (28) $\int_0^\infty \frac{e^x}{1+e^{2x}}dx = \lim_{b\to\infty} \int_0^b \frac{e^x}{1+e^{2x}}dx$ $\quad u = e^x \quad du = e^x dx$ $\quad$ u-limits: $0 \to 1$

$= \lim_{b\to\infty} \int_1^b \frac{du}{1+u^2} = \lim_{b\to\infty} \left[\arctan u\right]_1^b =$

$\lim_{b\to\infty} \left[\arctan b - \arctan 1\right] = \frac{\pi}{2} - \frac{\pi}{4}$

$= \boxed{\frac{\pi}{4}}$

(29) $\int_{-3}^{2} \frac{9}{x^2+x-20}dx = \boxed{-2.506}$ Calc (if non-calc, use partial fractions)

(30) (a) $f'(x) = x^2\ln x \quad f'(e) = e^2\ln e = e^2 = m$

equation of tangent @ $(e,2)$: $y-2 = e^2(x-e)$

(b) $f''(x) = 2x\ln x + x^2\left(\frac{1}{x}\right)$

$f''(x) = 2x\ln x + x$

☆tested # in interval $f''(2) = 4\ln 2 + 2$ = positive

∴ By 2nd der test, f is $\boxed{\text{concave up}}$ on int $1<x<3$ b/c f'' is + in this interval.

☆ yuk!!! (c) $f(x) = \int f'(x) = \int x^2\ln x\, dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad v = \frac{x^3}{3} \quad dv = x^2$

$\int x^2\ln x\, dx = \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2 dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C \rightarrow$ @ $(e,2) \rightarrow \frac{1}{3}e^3 - \frac{1}{9}e^3 + C = 2$

$C = 2 - \frac{2}{9}e^3$

$\boxed{\therefore f(x) = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + 2 - \frac{2}{9}e^3}$