CALCULUS BC
LAST AP REVIEW

Reminders: 1. Put new batteries in your calculator. Make sure your calculator is in **RADIAN** mode.
2. Get a good night's sleep Tuesday night.
3. Eat breakfast Wednesday morning.
4. Bring: TI-89, pencils, eraser, pen, watch, snacks, bottle of water, jacket.

Work these on **notebook paper**. Do not use your calculator unless the problem tells you to.

1. Evaluate $\lim\limits_{h\to 0}\dfrac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h}$

2. Evaluate: $\lim\limits_{x\to 2^-}\dfrac{|x-2|(x+5)}{x-2}$

3. Evaluate: $\int_0^3 |x-2|\,dx$

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4. Find a and b so that f is differentiable everywhere. $f(x)=\begin{cases} ax^2+1, & x\le 2 \\ bx-3, & x>2 \end{cases}$

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5. $f(x)=\begin{cases} -x, & x\le 1 \\ x^2-2x, & x>1 \end{cases}$ Evaluate: $\int_{-3}^{3} f(x)\,dx =$

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6. Let R be the region bounded by the graphs of $y=x^2$ and $y=x+6$.
(a) Find the area of R.
(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y=9$.
(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y=-1$.
(d) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the vertical line $x=5$.

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7. Let R be the region bounded by the graphs of $y=\sqrt{x}$ and $y=\dfrac{x}{2}$.
(a) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are rectangles whose heights are three times the lengths of the bases. Write, but do not evaluate, an integral expression for the volume of this solid.
(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are semicircles. Write, but do not evaluate, an integral expression that could be used to find the volume of this solid.

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8. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

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9. (Calculator) The first derivative of the function f is defined by $f'(x)=\sin\left(x^3-x\right)$ for $0\le x\le 2$. On what intervals is f increasing?

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10. (Calculator) The derivative of the function f is given by $f'(x)=x^2\cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?
(A) One (B) Two (C) Three (D) Four (E) Five

11. (Calculator) A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1+2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

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12. (Calculator) A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667 (B) 0.704 (C) 1.540 (D) 2.667 (E) 2.901

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13. **(2004 AB 3)** (Calculator) A particle moves along the y-axis so that its velocity at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

14.

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice-differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$. Evaluate $\int_2^{13} \left(3 - 5f'(x)\right)dx$. Show the work that leads to your answer.

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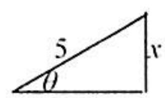
15. If θ increases at a constant rate of 3 rad/min, at what rate is x increasing in units/min when $x = 3$ units?



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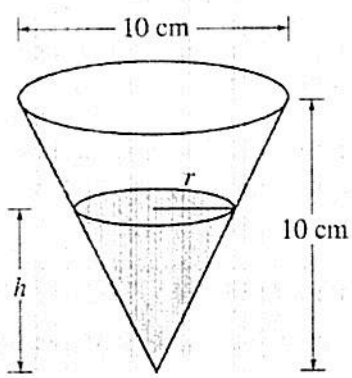
16. **(2002 AB 5)**



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

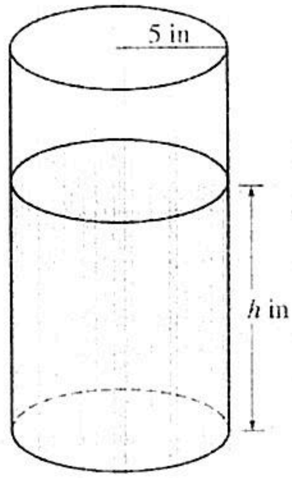
(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

17. **(2003 AB 5/BC 5)**



A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

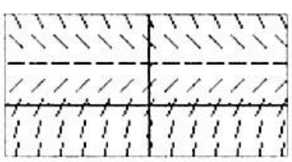
(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

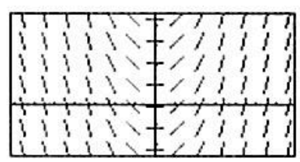
(c) At what time t is the coffeepot empty?

Match the slope fields with their differential equations.

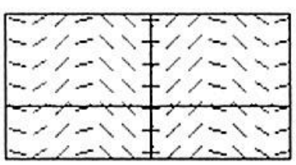
(A)



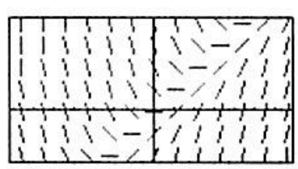
(B)



(C)



(D)



18. $\frac{dy}{dx} = \sin x$ 19. $\frac{dy}{dx} = x - y$ 20. $\frac{dy}{dx} = 2 - y$ 21. $\frac{dy}{dx} = x$

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22. Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 6x - 2xy$ where $f(0) = 7$.

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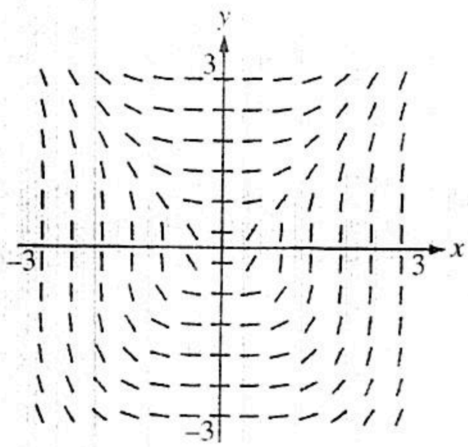
23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

(A) $y = 2x$ for $x > 0$ (B) $y = 2x - 6$ for $x \neq 3$ (C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$ (E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

24. **(2003 BC 14)**



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

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25. **(2005 Form B AB 2/BC 2)** (Calculator)

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate $W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right)$ gallons per hour. During the same time interval, water is removed from the tank at the rate $R(t) = 275\sin^2\left(\frac{t}{3}\right)$ gallons per hour.

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

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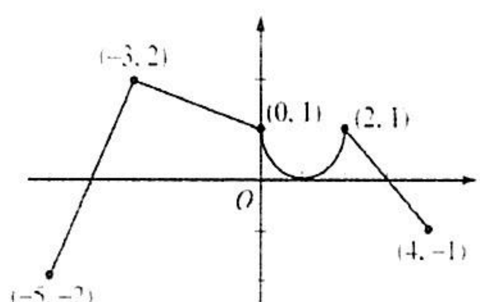
26. **(2004 AB 5)**

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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27. $\int x\sin(2x)\,dx =$

28. $\int \frac{dx}{x^2 - 6x + 8} =$

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29. If $f'(x) = 3x\,f(x)$ with $f(1) = 7$ and $\lim_{x\to\infty} f(x) = 2$, find $\int_1^{\infty} 3x\,f(x)\,dx$.

x	0	1
$f(x)$	2	4
$f'(x)$	6	−3
$g(x)$	−4	3
$g'(x)$	2	−1

30. **(2008 BC 22)**

The table above gives values of f, f', g, and g' for selected values of x. If $\int_0^1 f'(x)g(x)\,dx = 5$, then $\int_0^1 f(x)g'(x)\,dx =$

(A) -14 (B) -13 (C) -2 (D) 7 (E) 15

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31. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\,d\theta$ (B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)\,d\theta$ (C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\,d\theta$

(D) $\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\,d\theta$ (E) $\int_{7\pi/6}^{\pi/6} (1+2\sin\theta)\,d\theta$

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32. **(1998 BC 19)**

The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2\,d\theta$ (B) $\frac{1}{2}\int_{\pi/4}^{3\pi/4} (4\sin\theta - 2)^2\,d\theta$ (C) $\frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin\theta - 2)^2\,d\theta$

(D) $\frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(16\sin^2\theta - 4\right)d\theta$ (E) $\frac{1}{2}\int_0^{\pi} \left(16\sin^2\theta - 4\right)d\theta$

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33. Given the polar curves $r = 4\cos\theta$ and $r = 2$, write an integral expression which gives the common interior of $r = 4\cos\theta$ and $r = 2$

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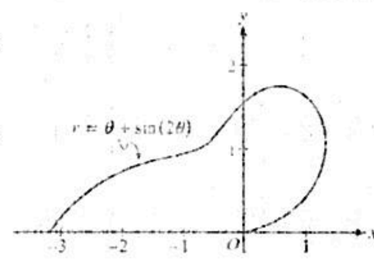
34. **(2005 BC 2)** (With calculator)

The curve on the right is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.



(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answers.

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35. Given the polar curve $r = 3 + 2\cos\theta$. Find the slope of the curve at the point where $\theta = \frac{\pi}{2}$.

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36. (Calculator) The rate of change, $\frac{dP}{dt}$, of the number of people at a dance who have heard a rumor is modeled by a logistic differential equation. There are 2000 people at the dance. At 9PM, the number of people who have heard the rumor is 400 and is increasing at a rate of 500 people per hour. Write a differential equation to model the situation.

37. The population $P(t)$ of fish in a lake satisfies the logistic differential equation $\frac{dP}{dt} = 3P - \frac{P^2}{6000}$.

(a) If $P(0) = 4000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(b) If $P(0) = 10{,}000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify.

(c) If $P(0) = 20{,}000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify.

(d) If $P(0) = 4000$, what is the population when it is growing the fastest? Justify your answer.

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38. Given $\frac{dy}{dx} = \frac{xy}{2}$. Let $f(x)$ be the particular solution to the given differential equation with initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$.

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39. **(2001 BC 1)** (Calculator) Given $\frac{dx}{dt} = \cos(t^3)$ and $\frac{dy}{dt} = 3\sin(t^2)$ for $0 \le t \le 3$. At time $t = 2$, the object is at position (4, 5).

(a) Find the speed and the acceleration vector at time $t = 2$.

(b) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(c) Find the position of the object at time $t = 3$.

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40. **(2001 BC 6)** A function is defined by $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \ldots + \frac{(n+1)}{3^{n+1}}x^n + \ldots$ for all x in the interval of convergence of the given power series.

(a) Find $\lim_{x\to 0} \dfrac{f(x) - \frac{1}{3}}{x}$.

(b) Write the first three nonzero terms and the general term for the infinite series that represents $\int_0^1 f(x)\,dx$.

(c) Find the sum of the series found in part (b).

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41. If f is a function such that $f'(x) = \sin(x^2)$, then the coefficient of x^7 in the Maclaurin series for $f(x)$ is ?

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42. The coefficient of x^3 in the Taylor series for e^{2x} about $x = 0$ is ?

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43. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converge?

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44. **(1998 BC 18)** Which of the following converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

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45. (Calculator) The function f has derivatives of all orders for all real numbers x. Assume that $f(2) = 5$, $f'(2) = -3$, $f''(2) = 4$, $f'''(2) = -1$, and $\left|f^{(4)}(x)\right| \le 3$ for all x in [1.57, 2].

(a) Write the third-degree Taylor polynomial for f about $x = 2$.

(b) Use your answer to (a) to approximate $f(1.57)$. Give your answer correct to five decimal places.

(c) Use the Lagrange error bound on the approximation of $f(1.57)$ to explain why $f(1.57) \ne 6.8$.

46. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in its interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n (n+2)}$ and $f(5) = \dfrac{1}{2}$. Show that the first four terms of the Taylor polynomial for f about $x = 5$ approximates $f(6)$ with an error less than $\dfrac{1}{50}$. Justify your answer.

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47. A function f has Maclaurin series given by $\dfrac{x^4}{2!} + \dfrac{x^5}{3!} + \dfrac{x^6}{4!} + \ldots + \dfrac{x^{n+3}}{(n+1)!} + \ldots$. Which of the following is an expression for $f(x)$?

(A) $-3x\sin x + 3x^2$ (B) $-\cos(x^2) + 1$ (C) $-x^2 \cos x + x^2$ (D) $x^2 e^x - x^3 - x^2$ (E) $e^{x^2} - x^2 - 1$

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48. What is the value of $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$?

49. What is the value of $\displaystyle\sum_{n=0}^{\infty} 5(\cos x)^n$ if $x = \dfrac{\pi}{3}$?

50. Find the sum of $1 + \sin 2 + \dfrac{\sin^2 2}{2!} + \dfrac{\sin^3 2}{3!} + \ldots + \dfrac{(\sin 2)^n}{n!} + \ldots$

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51. (a) Find a power series for $f(x) = \dfrac{1}{1-x^2}$ centered at $x = 0$. Write the first four nonzero terms and the general term. For what values of x does this series converge?

(b) Find a power series for $f'(x)$. Write the first four nonzero terms and the general term.

(c) Use your answer to (b) to find the sum of $\dfrac{2}{3} + \dfrac{4}{27} + \dfrac{6}{243} + \ldots + 2n\left(\dfrac{1}{3}\right)^{2n-1} + \ldots$ if possible. If it isn't possible, explain.

(d) Use your answer to (b) to find the sum of $2\left(\dfrac{4}{3}\right) + 4\left(\dfrac{4}{3}\right)^3 + 6\left(\dfrac{4}{3}\right)^5 + \ldots + 2n\left(\dfrac{4}{3}\right)^{2n-1} + \ldots$ if possible. If it isn't possible, explain.

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52. (a) Find a power series for $f(x) = \dfrac{1}{1+x}$ centered at $x = 0$. Write the first four nonzero terms and the general term. For what values of x does this series converge?

(b) Find a power series for $\int_0^x f(t)\,dt$. Write the first four nonzero terms and the general term.

(c) Use your answer to (b) to find the sum of $\dfrac{1}{2} - \dfrac{\left(\frac{1}{2}\right)^2}{2} + \dfrac{\left(\frac{1}{2}\right)^3}{3} - \dfrac{\left(\frac{1}{2}\right)^4}{4} + \ldots + (-1)^{n+1}\dfrac{\left(\frac{1}{2}\right)^n}{n} + \ldots$ if possible.