* Calc # 56 + 60

AP Review 6

56. $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$ @ $t=0$, Position is at $x=1$

a) $a(t) = v'(t)$

$a(2) = v'(2) = 1.588$ (positive)

$v(2) = -(2+1)\sin\left(\frac{2^2}{2}\right) = -2.728$ (negative)

The speed is decreasing @ $t=2$ because velocity & acceleration have opposite signs.

b) $v(t)=0$ @ $t=2.507$ over $(0,3)$

Particle changes direction at $t=2.507$ since this is were velocity changes signs (neg to pos).

c) $\int_0^{2.507} v(t)\,dt = -3.265$ Sto A $\quad \int_{2.507}^{3} v(t)\,dt = 1.068$ Sto B

total dist traveled from $t=0$ to $t=3$ is $\boxed{4.334}$

d) Greatest dist $[0,3]$ occurs @ $t=2.507$

$S(t) = \int_0^{2.507} v(t) = S(2.507) - S(0) \rightarrow S(0)=1$

$-3.265 = S(2.507) - 1$

$S(2.507) = -2.265$

Greatest distance from origin is $|-2.265| = 2.265$

E 57. $g(x) > 0$ for all x

$f(0) = 1$

$h(x) = f(x)g(x) \quad h'(x) = f(x)g'(x) \quad f(x) = ?$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$f'(x) = 0$, so $f(x) =$ constant $= 1$

D 58. area approx of each piece ≈ 3000

D 59. $f'(x) = \dfrac{2x(x-1) - (x^2-2)}{(x-1)^2}$

$= \dfrac{x^2 - 2x + 2}{(x-1)^2}$ $\boxed{f'(2) = 2}$

60(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50-40} = \dfrac{55-40}{10} = \boxed{1.5 \text{ gal/min}^2}$

(b) $R''(45) = 0$ (P.O.I) since $R'(t)$ is a maximum @ $t=45$, & $R'(t)$ is differentiable

(c) $\int_0^{90} R(t)\,dt \approx 30(20) + 10(30) + (10)40 + 20(55) + 20(65)$

$\approx \boxed{3700 \text{ gallons of fuel}}$

(d) $\int_0^b R(t)\,dt$ gives the total gallons of fuel consumed in the first b minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ gives the average value of the rate of the rate of fuel consumption in gal/min during first b minutes

A 61. f(x) linear, f'(x) constant, $f''(x) = 0$, so $\int f''(x) = 0$

D 62. $F(x) = \int_0^x \sqrt{t^3+1}\,dt$ (2nd FTC) $= \sqrt{x^3+1}$ @ 2 $= \sqrt{9} = \boxed{3}$

E 63. $f(x) = \sin(e^{-x})$ $f'(x) = \cos(e^{-x})(-1)(e^{-x})$

$= -e^{-x}\cos(e^{-x})$

C 64. $f''(x) = 0$ @ $x = 0, -1, 2$

+ | − | + | + POI @ x = −1 & 0

−1 0 2

A 65. $\int_{-3}^{K} x^2\,dx = \left.\dfrac{x^3}{3}\right|_{-3}^{K} = \dfrac{K^3}{3} - \dfrac{(-27)}{3}$

$\dfrac{K^3}{3} + 9 = 0$ $\dfrac{K^3}{3} = -9$ $\boxed{K = -3}$