

AP REVIEW 6

Work these on notebook paper. Use your calculator on problems 56 and 60.

56. A particle moves along the x -axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
 (b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.
 (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.
 (d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

57. Let f and g be differentiable functions with the following properties:

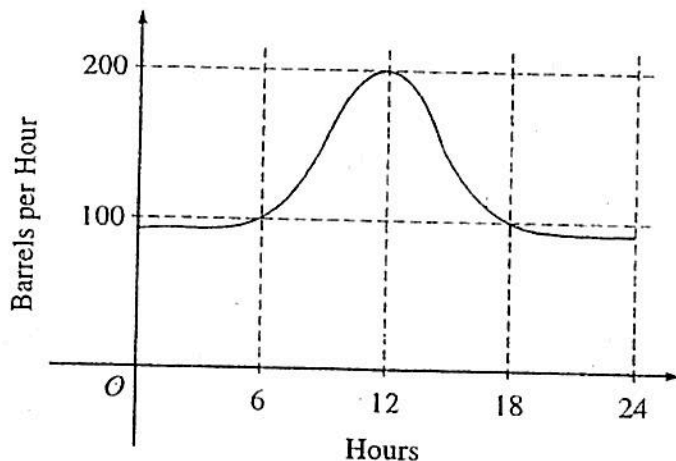
(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

- (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

- 58.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

- (A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

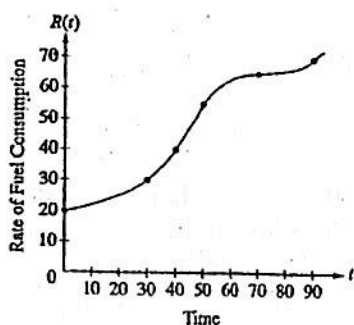
59. What is the instantaneous rate of change at $x = 2$ of the function f given

by $f(x) = \frac{x^2 - 2}{x - 1}$?

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

60.

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

61. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$

- (A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

62. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

- (A) -3 (B) -2 (C) 2 (D) 3 (E) 18

63. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

- (A) $-\cos(e^{-x})$ (B) $\cos(e^{-x}) + e^{-x}$ (C) $\cos(e^{-x}) - e^{-x}$
 (D) $e^{-x} \cos(e^{-x})$ (E) $-e^{-x} \cos(e^{-x})$

64. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

- (A) -1 only (B) 2 only (C) -1 and 0 only
 (D) -1 and 2 only (E) -1, 0, and 2 only

65. What are all the values of k for which $\int_{-3}^k x^2 dx = 0$?

- (A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3