AP REVIEW 4

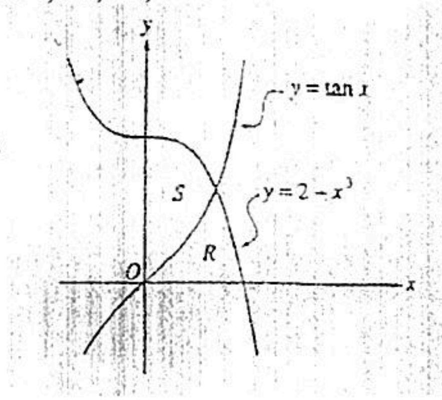
Work these on notebook paper. Use your calculator only on problems 34, 39, 43, and 45.

34. Let R and S be the regions in the first quadrant shown in the figure on the right. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.



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35. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

(A) None (B) I only (C) III only (D) I and III only (E) II and III only

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36. $\int \frac{dx}{(x-1)(x+3)} =$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$ (B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$ (C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$ (E) $\ln\left|(x-1)(x+3)\right| + C$

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37. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$ (B) $\int_0^2 (2-y)\,dy$ (C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$ (E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

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38. The $\lim_{h\to 0} \frac{\tan(3(x+h)) - \tan(3x)}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

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39. Let $F(x) = \cos(2x) + e^{-x}$. For what value of x on the interval [0, 3] will F have the same instantaneous rate of change as the average rate of change of F over the interval?

(A) 1.542 (B) 1.610 (C) 1.678 (D) 1.746 (E) 1.814

40. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.

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41. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

42.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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43. If $f'(x) = \frac{x^2}{1+x^5}$ and $f(1) = 3$, then $f(4) =$

(A) 2.988 (B) 3 (C) 3.016 (D) 3.376 (E) 3.629

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44. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$ (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ (C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$ (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

45.

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

The table above gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$. What is this estimate?

(A) 1,910 gal (B) 14,100 gal (C) 16,930 gal (D) 18,725 gal (E) 20,520 gal