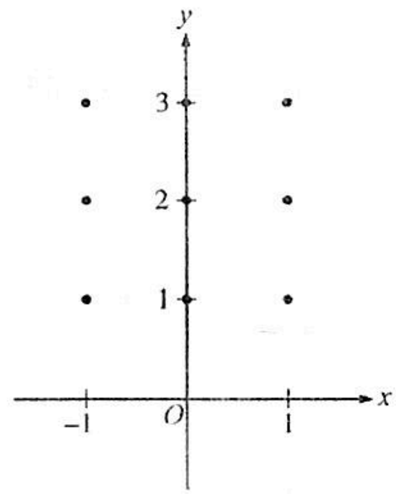
AP REVIEW 3

Work these on notebook paper (except for 25(a)). **No calculator**.

25. Consider the differential equation given by $\frac{dy}{dx} = xy$.



(a) On the axes provided, sketch a slope field for the given differential equation at the 12 points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 5$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 5$. Then use your solution to find the exact value of $f(0.2)$.

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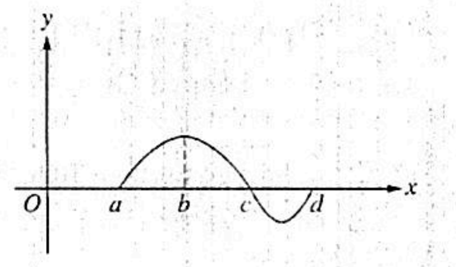
26. The graph of f is shown in the figure on the right. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?



(A) a (B) b (C) c (D) d

(E) It cannot be determined from the information given.

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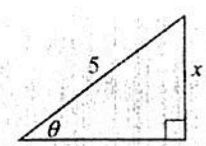
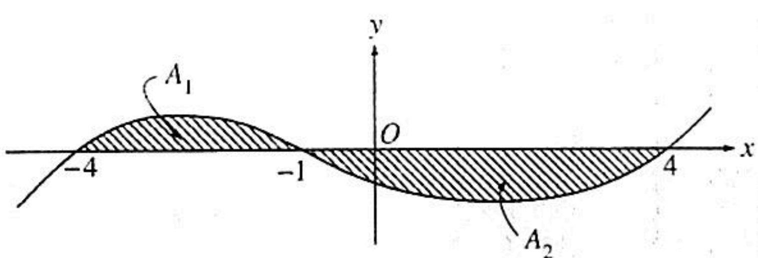
27. In the triangle shown on the right, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when $x = 3$ units?



(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

28.



The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

29. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

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30. $\lim\limits_{h \to 0} \dfrac{\ln(e+h) - 1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$

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31. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \dfrac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

(D) $f(c) = 1$ for at least one c between -3 and 6

(E) $f(c) = 0$ for at least one c between -1 and 3

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32. If $\dfrac{dy}{dx} = (1 + \ln x)y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1 + \ln x$

(C) $\ln x$

(D) $e^{2x + x\ln x - 2}$

(E) $e^{x \ln x}$

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33. $\int x^2 \sin x \, dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$

(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$

(C) $-x^2 \cos x + 2x \sin x + 2\cos x + C$

(D) $-\dfrac{x^3}{3} \cos x + C$

(E) $2x \cos x + C$