AP Calculus - BC Name ___________________

3.7 Related Rates

Questions that ask for the calculation of the rate at which one variable changes, based on the rate at which another variable is known to change, are called related rates.

A "rate of change" is referring to an instantaneous rate of change, which is a derivative.

Practice translating sentences into proper calculus notation.

1) The area of a circle is increasing at a rate of six square inches per minute.

What is changing with respect to time?_____

How can we write this using derivative notation?

$$\frac{d\,\_\_\_}{dt} = \_\_\_\_\_$$

2) The volume of a cone is decreasing at a rate of two cubic feet per second.

What is changing with respect to time?_____

How can we write this using derivative notation?

$$\frac{d\,\_\_\_}{dt} = \_\_\_\_\_$$

3) The height of a tree is increasing at a rate of 1/2 foot per year.

What is changing with respect to time?_____

How can we write this using derivative notation?

$$\frac{d\,\_\_\_}{dt} = \_\_\_\_\_$$

4) The water level in my fish tank is decreasing at a rate of 2 inches per hour.

What is changing with respect to time?_____

How can we write this using derivative notation?

$$\frac{d\,\_\_\_}{dt} = \_\_\_\_\_$$

Related rate problems require you to take a derivative with respect to time.
Differentiate each equation with respect to time. (You will need to use implicit differentiation)

1) $A = \pi r^2$

2) $C = 2\pi r$

3) $V = s^3$

4) $V = \pi r^2 h$

5) $a^2 + b^2 = c^2$

6) $A = ½ bh$

1) Suppose that x and y are both differentiable functions of t and are related by the equation $y = x^2 + 3$. Find $\frac{dy}{dt}$ when x = 1, given that $\frac{dx}{dt} = 2$ when x = 1.

Steps for Related Rates

1) Draw a picture. Draw several so that the situation can be seen at more than one instant.
2) Identify the variable whose rate of change you seek.
3) Find a formula relating the variables whose rate of change you seek with one or more variables whose rate of change you know.
4) Differentiate implicitly with respect to time t.
5) Substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.
6) Make sure that you have answered the question asked. It is a good idea to write your answer in a complete sentence.

2) A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

3) Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

4) Let A be the area of a square whose sides have length x, and assume x varies with time. At a certain instant the sides are 3 feet long and growing at a rate of 2 ft/min. How fast is the area growing at that instant?

5) Let V be the volume of a cylinder having height h and radius r, and assume that h and r vary with time. At a certain instant, the height is 6 inches and increasing at 1 in/sec while the radius is 10 inches and decreasing at 1 in/sec. How fast is the volume changing at that instant? Is the volume increasing or decreasing?

6) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How rapidly is the area enclosed by the ripple increasing at the end of 10 seconds?

7) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 miles squared per hour. How fast is the radius of the spill increasing when the area is 9 miles squared?

8) A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?