AP Calculus Name ______________________

3.6 Inverse Functions NOTES

Function f(x) = x + 3 can be represented by a set of ordered pairs.

f: {(1,4), (2, 5), (3, 6), (4, 7)}

By interchanging the first and second coordinates of each ordered pair, you can form the inverse function of f.

f^{-1}:

The domain of f = the ________________ of f^{-1}.

The range of f = the _________________ of f^{-1}.

The functions f and f^{-1} have the effect of "undoing" each other.

Explain how to "undo" each of the following functions. Then write the inverse function of f.

1) f(x) = x – 5 2) f(x) = $\frac{x}{2}$ 3) f(x) = 3x + 2 4) f(x) = x^3

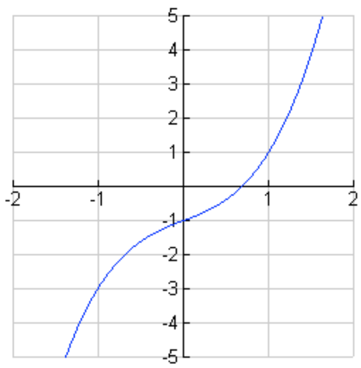
<u>Definition of Inverse Functions</u>
A function g is the inverse function of the function f if **f(g(x)) = x** for each x in the domain of g and **g(f(x)) = x** for each x in the domain of f.

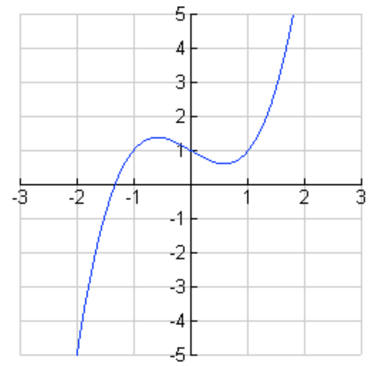
5) Show that f(x) = $2x^3 - 1$ and g(x) = $\sqrt[3]{\frac{x+1}{2}}$ are inverses of each other.

The Existence of an Inverse Function

1) A function has an inverse function if and only if it is one-to-one (passes the Horizontal Line Test).

2) If f is strictly monotonic on its entire domain (either increases or decreases on the entire domain), then it is one-to-one and therefore has an inverse function.





Guidelines for Finding an Inverse Function

1) Use the **HLT** to determine whether the function given by y = f(x) has an inverse function.

2) Interchange x and y.

3) Solve the equation for y. The resulting equation is $y = f^{-1}(x)$.

4) Define the domain of f^{-1} to be the range of f.

6) Find the inverse function of $f(x) = \sqrt{2x - 3}$.

7) Use the derivative to determine whether the function $f(x) = x^3 - 6x^2 + 12x$ is strictly monotonic on its entire domain and therefore has an inverse function.

Derivative of an Inverse Function

8) Consider the function $f(x) = x^3$.

Calculate the slope of f at (1, 1), (2, 8), and (3, 27).

Find the inverse of f(x) (name the inverse g(x)).

Calculate the slope of g at (1, 1), (8, 2) and (27, 3).

What do you observe?

Steps to find the derivative of an inverse function (g(x)) at a point (a, b):

1) Make an f and $g = f^{-1}$ column on your paper.
2) Define the points of f and g.
3) In the f column, write the function f(x).
4) Find the derivative of f(x).
5) Find the slope of f at its defined point.
6) Find the slope of g at its defined point, by taking the reciprocal of the slope of f.

9) Let f be the function defined by $f(x) = \sqrt{x-4}$. If $g(x) = f^{-1}(x)$ and $g(1) = 5$, what is the value of $g'(1)$?

10) Let f be the function defined by $f(x) = x^3 + 2x - 1$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

11) Let f be the function defined by $f(x) = \dfrac{x+3}{x-5}$. If $g(x) = f^{-1}(x)$ and $g(-7) = 4$, what is the value of $g'(-7)$?

Homework: Worksheet