

### 2.3 Product and Quotient Rules and Higher-Order Derivatives

#### The Product Rule

The derivative of  $f \cdot g$  is the second function times the derivative of the first, plus the first function times the derivative of the second.

$$\frac{d}{dx}[f(x) \cdot g(x)] =$$

Find the derivative.

1)  $h(x) = (3x - 2x^2)(5 + 4x)$

2)  $f(x) = (6x + 5)(x^3 - 2)$

3)  $g(s) = \sqrt{s}(4 - s^2)$

4)  $y = 3x^2 \sin x$

5)  $y = 2x \cos x - 2 \sin x$

6)  $g(x) = \sqrt{x} \sin x$

#### The Quotient Rule

The derivative of  $\frac{f}{g}$  is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] =$$

Find the derivative.

7)  $y = \frac{5x - 2}{x^2 + 1}$

8)  $g(x) = \frac{x^2 + 2}{2x - 7}$

9)  $f(x) = \frac{\cos x}{x^3}$

10)  $h(x) = \frac{x}{\sqrt{x} - 1}$

11) Find an equation of the tangent line to the graph of  $f(x) = \frac{3 - x}{x + 5}$  at  $(-1, 1)$ .

Find the derivative. (the difference in the quotient rule and the constant multiple rule)

$$12) y = \frac{x^2 + 3x}{6}$$

$$13) y = \frac{5x^4}{8}$$

$$14) y = \frac{-3(3x - 2x^2)}{7x}$$

$$15) y = \frac{9}{5x^2}$$

### Derivatives of Trig Functions

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Find the derivative.

$$16) y = x - \tan x$$

$$17) y = x \cdot \sec x$$

$$18) y = \frac{1 - \cos x}{\sin x}$$

$$19) y = \csc x - \cot x$$

### Higher-Order Derivatives

$s(t)$  position function

$v(t) = s'(t)$  velocity function

$a(t) = v'(t) = s''(t)$  acceleration function

20) An automobile's velocity starting from rest is  $v(t) = \frac{100t}{2t + 15}$  where  $v$  is measured in feet per second. Find the acceleration function and then find the acceleration at  $t = 5$ .

21) A car is traveling at a rate of 66 feet per second (45 miles per hour) when the brakes are applied. The position function for the car is  $s(t) = -8.25t^2 + 66t$ , where  $s$  is measured in feet and  $t$  is measured in seconds. Find the car's position, the car's velocity, and the car's acceleration at  $t = 2$  seconds.

Homework: Page 126 – 129 # 3, 7, 13, 17, 25, 41, 53, 63, 67, 71, 93, 97, 101, 109, 111, 115 (16 problems)