AP Calculus Name ________________________

3.1 The Derivative and the Tangent Line Problem

<u>Slope</u>

f(x) = 2x + 3

$f(x) = x^2 + 2$

<u>Slope of a Tangent Line</u>

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

1) Find the slopes of the tangent lines to the graph of $f(x) = x^2 + 1$ at the points (0, 1) and (-1, 2).

<u>Limit Definition of a Derivative</u>

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$ or $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of a function of x is also a function of x. This "new" function gives the slope of the tangent line to the graph of f at the point (x, f(x)), provided that the graph has a tangent line at this point.

The process of finding the derivative of a function is called <u>differentiation</u>. A function is <u>differentiable</u> at x if its derivative exists at x.

f'(x) is read "f prime of x".

Notations for derivative: f'(x) y' $\frac{dy}{dx}$ $\frac{d}{dx}[f(x)]$

2) Find the derivative of $f(x) = x^2 + 2x$ using the limit definition of a derivative.

3) $f(x) = 2x^2 - 3$

a) Find the derivative of f(x) using the limit definition of a derivative.

b) Find the slope of f(x) at the point (1, 2).

c) Find the equation of the tangent line to the graph of f at the point (1, 2).

4) What is f(x)? (In other words, what function is this expression finding the derivative of?)

a) $f'(x) = \lim_{h \to 0} \frac{[(x+h)+2]-(x+2)}{h}$

b) $f'(x) = \lim_{h \to 0} \frac{[3(x+h)^2+5]-(3x^2+5)}{h}$

c) $f'(x) = \lim_{h \to 0} \frac{[2(x+h)^3-3(x+h)+7]-(2x^3-3x+7)}{h}$

d) $f'(x) = \lim_{h \to 0} \frac{[5\sqrt{x+h}-3]-(5\sqrt{x}-3)}{h}$

<u>Alternative Form of Derivative</u> (used to find the slope at a specific x-value)

$$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

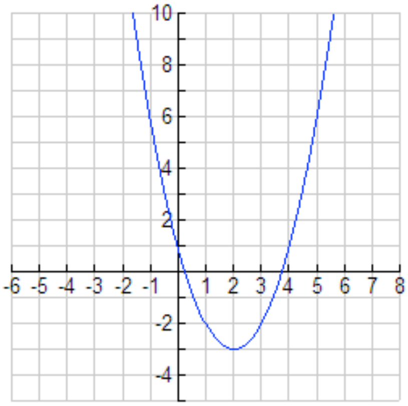
5) Find the slope of $f(x) = x^2 + 3$ at the point (2, 7) using the alternative form of a derivative. Then write the equation of the tangent line at that point.

6) What would this equation $f'(5) = \lim_{x \to 5} \frac{(3x^2-2x+1)-66}{x-5}$ be used to find?

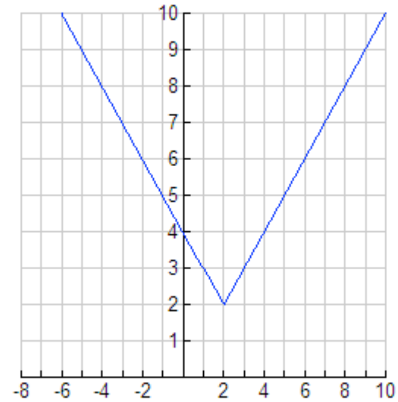
<u>Differentiability and Continuity</u>

1) If a function is differentiable at x = c, then it is continuous at x = c. So, <u>differentiability implies continuity</u>.
2) It is possible for a function to be continuous at x = c and not be differentiable at x = c. So, <u>continuity does not imply differentiability</u>.

Graph with a smooth turn.

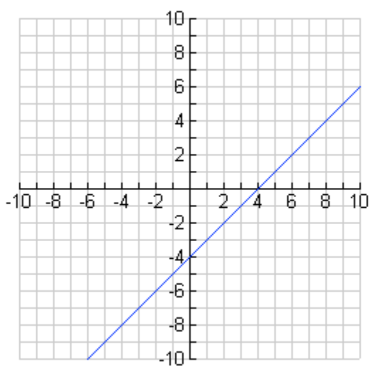


Graph with a sharp turn.



Sketch the graph of f'.

7)



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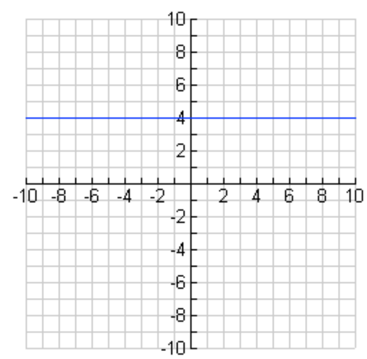

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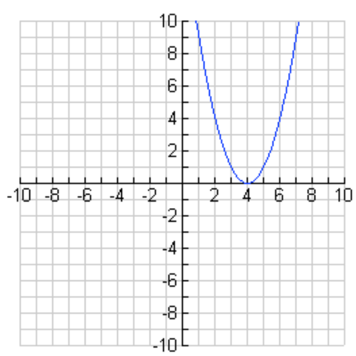

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